DYNAMICAL BIFURCATION: 
DIFFUSION OF CRIME ON LABOR FORCE

Yun Hariadi  
(faust_doktor@yahoo.com)  
Department of Dynamical System Modeling  
Bandung Fe Institute

Abstract
This paper studies diffusion of crime on labor force in two-way causality approach crime and 
unemployment. The diffusion model of this paper uses epidemical SIRS advanced in Pearl-
Verhulst map with time delay T. We found that the number of crimes will be fluctuating to be 
chaos in change control parameter r_0 on dynamical bifurcation map, this transition from order to 
disorder will disappear period doubling and reach chaos faster. The crime-attractor will appear 
on chaos region if delta time iteration \( \Delta t \) less from one and great time delay T.

Key words
Crime, unemployment, diffusion, Pearl-Verhulst map, dynamical bifurcation, chaos, crime-
attractor.

Introduction
Chaos theory is another side of non-linear dynamical system giving new view on 
understanding in social problems. The problems on social sciences are dynamic and full of 
uncertainty; it is hard to predict the behavior of system. Chaos theory gives possibility to study 
the behavior of the system in its transition from order to disorder that caused by small change 
on control parameter.

Crime as part of the behavior social appears complex behavior, i.e.: how crime for the 
first time suddenly appears and how the number of crime unexpectedly fluctuating.

Corporate crime will increase vast on small change in an external variables, say tax 
rates, a small change in unemployment rates may kick in a large change in both property crimes 
and domestic violence, white collar crimes may increase with small change in the budget a 
manager, price and availability of oil is the variables of choice in International crime waves today 
(Young, T.R, 1989).

Crime in chaos theory as social system as a non-linear dynamical system has properties 
like sensitivity on initial conditions that small change in control parameter will caused great 
change on the behavior of crime.

Crime behavior on labor market has been known before. Employment and 
unemployment as agent on market labor have probability and chance to commit crime. Both 
employment and unemployment tends to do crime for greater payoff (Burdett et al, 2002).

Pay-off of the employment is the function of wages, and the pay-off of the 
unemployment payoff is the function of insurance and savings. Unemployment is defined as 
agent not working in current period and search for new production opportunity (Gomes et al, 
2001).

The employment in the working period and the unemployment searching for job or in no-
job period have choices to increasing their payoff, i.e.: crimes. The pay-off of the crime is the 
function of money, law-enforcement \( \pi \) and lucky g.

The payoff of the employment \( V_1(w) \), the payoff of the unemployment \( V_0 \), and the payoff 
in jail J. Thus, the payoff of the crime for unemployment is said to be \( K_0=g+\pi+(1-\pi)V_0 \) and for 
the employment \( K_1(w)=g+\pi+(1-\pi)V_1(w) \), hence each agent will commit to crime for \( K_0>V_0 \) and 
\( K_1(w)>V_1(w) \). (Burdett et al, 2002).

Employment, unemployment, and crimes make social relationship. Each agent is 
connected with other agent on social network transmitting information. The information network 
is considered to be the important factor that change state agent to another state.

Let U representing the number of unemployment, C representing the number of crimes, 
and s representing the network size for information. The agent that commit and doing crime 
tends to limit their information network by decreasing the network size generally. Increasing of
the rate crime brings to the decreasing of the network size $\delta s/\delta C<0$, while the decreasing of the network size will then increasing rate unemployment $\delta U/\delta s<0$. Therefore, the increasing rate crime will increase the rate of the unemployment $\delta U/\delta C>0$ (Calvo-Zenou, 2002).

**The Diffusion of Crime**

Two-way causality approaching the crime-unemployment have emerged an interesting question: where does the increasing of the unemployment rate come from? Naturally, the unemployment grows a long with the growing labor force which is not participating in the labor market. The increasing rate crime influences the chances and the opportunity to getting job by decreasing it. This occurs because the decreasing of the network size and the stigma on the increasing rate crime (labeling theory of crime).

The interaction among agents in the social network is an important factor to construct the model of the social network. In social network model, information diffused contains payoff and chances on changing from one state to other. Increasing the intensity of the interaction shown by the increasing information transmitted will hike the chance to changing their state. These propositions are known in the differential association theory as a theory of crime.

The crime rate – in the two-way causality approach on crime-unemployment - will increase the unemployment rate that finally increase the crime rate itself. In the other words, crime diffuses to labor force as choice on effort to increasing the payoff. Diffusion of crime on labor force is modeled by observing the unemployment and employment as labor force committing crime. This is guaranteed by two-way causality. At least there are flows from state of unemployment to the state of crimes. In the other side, crime is guaranteed by two-way causality flowing to the state of the unemployment; and so on while this cycle will turn on making the closing cycle.

![Diagram](fig. 1)

The bold arrow shows the two-way causality of crime-unemployment

where $S_1(t)$, $S_2(t)$, and $I(t)$ representing the number of employment, unemployment, and crime at time $t$, respectively. The $a$ and $c$ represents the rate of crime as flows from employment and the rate of crime as flow from unemployment, respectively. The $b$ represents the rate of unemployment as flows from crimes, needing the time delay $T$ for labor force to reaching the crime state. The $d$ represents rate of employment as flows from unemployment and, the $e$ represents rate of unemployment as flows from employment.

\[
\begin{align*}
\frac{dS_1(t)}{dt} &= -aI(t-T)S_1(t-T) - eS_1(t) + dS_2(t) \\
\frac{dS_2(t)}{dt} &= -cI(t-T)S_2(t-T) + bI(t) + eS_1(t) - dS_2(t) \\
\frac{dI(t)}{dt} &= aI(t-T)S_1(t-T) + cI(t-T)S_2(t-T) - bI(t)
\end{align*}
\]

\[d(t)=S_2(t)/S_1(t)\]

\[
dI(t)/dt= r(t)I(t-T)[K-I(t-T)]-bI(t), \quad r(t) = \frac{[ad(t)+c]}{[d(t)+1]},
\]

which is on the discrete model known as Pearl-Verhulst map:

\[I(t+\Delta t)=I(t)+ \Delta t \ r(t) \ I(t-T)[K-I(t-T)]-\Delta t \ b \ I(t)\]
The Simulation

Simulation for variety of $\Delta t$ and $T$ resulting below by using the constant

\[
\begin{align*}
I(0) &= 0.1, \\
b &= 1, \text{ and} \\
K &= 1 \\
r(i+1) &= r(i) + 0.00001
\end{align*}
\]

BIFURCATION MAP

Bifurcation map shows the stability-number of crimes (attractor) on 200 last iterations from 1000 iterations. It moves from order to disorder by changing the control parameter $r_0$. It also moves from one point attractor to N-point attractor. For N limit to infinite is the chaos region.

[Figure A.1]

In this case, attractor moves from single point attractor to another attractor with period doubling. For the fist time, chaos is reached in $r_0=3.5$ and in the chaos region ($r_0>3.5$) there will be an order region, with time interaction that shown by one unit iteration and time delay $T=0$.

[Figure A.2 & A.3]

In this case, the intensity of the interaction among agents arises more than in the case 1. This represents the information exchange is arising, and agents in labor force committing to crime will do crime on the one next period. Comparing this case with the first, chaos region is reached in small parameter control, $r_0=3.1$, and the period doubling appeared in this case will not over than two-point attractors. This represents that number of crime chaotically fluctuating in small parameter control although in the next parameter control its fluctuation will tend to order.

[Figure A.4 & A.5]

Here, the intensity of the interaction among agents arises more than in the 2nd case. This behavior is seemingly to be the radical changes shown by disappearance of the period doubling. The number of crime-attractor changes from one-point attractor straight to the strange attractor. Thus, in the 4th & the 5th cases, it differs by time delay in one period influencing the transition from disorder to order. In the 4th case the order in disorder appears to be longer on parameter control than in the 5th. This fact shows that long time delay influences to keep the condition of the disorder.

[Figure A.6, A.7, A.8, A.9, A.10, & A.11]

This case is not different with the before one. The increasing of the intensity of the interaction which is far from one unit of iteration causes the disappearance of the period doubling. Thus the chaotic condition will be reached by small control parameter. The more length of the time delay caused the smaller control parameter to reaching chaotic condition.

POINCARE MAP

This simulation shows the direction of the fluctuating number of crime in chaotic region. This simulation maps the number of crime at time $t$ to the number of crime at time $t+T$, or formally $I(t)\text{vs}I(t+T)$. This will give the information about the number of crime doing crime on the next $T$ period by the number of crime at this time. Each case will be simulated at the chaotic region, $r_0=3.9$ by 1000 iterations.

In the first six pictures, it is shown that the direction of the fluctuating number of crime moves toward order that give us possibility to predict the point attractor. The longer time delay $T$ influences the spreading of the crime attractor on the Poincare map. This fact gives information about the faster agent to work on crime - within the intensity of interaction increasing – causes the impossibility to predict the crime attractor.
In the four last pictures, it is shown that the direction of the fluctuating number of crime moves toward the more order than figured in the first six picture above. Indeed in this case, the crime attractor moves to the center and builds the strange attractor ergodically.

Transition of the Poincare map figured from the first up to the last moves toward the more order conditions. This gives us any possibility to knowing the directions and the behaviors of the crime attractor. The increasing of the interaction intensity and the hiking of the time delay makes the number of crime chaotically fluctuates, but at chaos region the Poincare map shows the strange attractor ergodically.

Conclusion

The diffusion of the crime on labor force with two-way causality crime-unemployment seems to the occurrence of the chaos. The number of crime fluctuates from order to disorder, indeed the transition does not pass through the period doubling in the dynamical bifurcation map.

The increasing of the interaction intensity and the time delay makes the number of crime chaotically fluctuating, and the chaotic region will be reached by small parameter control.

At the chaotic region the increasing of the interaction intensity and time delay makes the crime attractor moves straight to center building the strange attractor ergodically.

Further Work

The further work of this paper can be gained by developing the model of crime diffusion by using the variation of the network information size and the time delays, flows of agent from one state to another with stochastic process.

Acknowledgment

The simulation resulted in this journal is made in the platform of MATLAB R.5.1.1. Privately, it is a very grace effort to publish the journal in English. The author realizes this as a handicapped to be overcome in the future, as soon as possible. For the translation to the good English, the author thanks for the generosity of Hokky Situngkir for accomplishment.

References

APPENDIX
THE LIST OF FIGURES

A. SIMULATION OF THE BIFURCATION MAP

Figure A.1

Figure A.2

Figure A.3

Figure A.4

Figure A.5
B. SIMULATION OF THE POINCARE MAP

Figure B.1

Figure B.2

Figure B.3

Figure B.4

Figure B.5

Figure B.6