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The Design of Desired Collectives with Agent-based Simulation

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Abstract
In this paper I consider the issue realizing efficient and equitable utilization of limited resources by collective decision of interacting heterogeneous agents. There is no presumption that collective action of interacting agents leads to collectively satisfactory results without any central authority. How well agents do for it in adapting to their environment is not the same thing as how satisfactory an environment they collectively create. Agents normally react to others' decisions, and the resulting volatile collective decision is often far from being efficient. By means of agent-based simulation, we explore a new approach for designing desirable collective of interacting agents. There are two closely related issues concerning collective, the forward problem and the inverse problem. The forward problem is to investigate what a collective of interacting agents determines its complex emergent behavior and therefore its performance. The inverse problem is to investigate how each agent should perform to induce a desirable performance as a whole. We discuss how agent-based simulation contributes to solve these two issues.
Keywords: forward problem, inverse problem, congestion, collective learning, coupling, efficiency, equity

1. Introduction

Today we have many challenges for designing large-scale and complex systems consisting of multiple physically and geographically distributed processors. Such systems offer the promise of speed, reliability, extensibility and the potential for increased tolerance to uncertain date and knowledge. The conventional approach requires a unifying control or coordination mechanism in order to extend the partial views and the incomplete knowledge of components and to guide a global solution. By true meaning of distribution that both control and knowledge are physically and often geographically distributed, we mean that there is neither global control nor global knowledge storage. We need to explore some methodologies for designing systems without central authority. There are two technologies that need to be developed to address this view of complex systems. First, distributed coordination mechanisms that enable systems to organize themselves and accomplish critical tasks with available resources. Second, self-organizing techniques that enable systems to improve with experience themselves.

The design of efficient collectives from bottom up becomes to be important issues in many areas (Duffy & Hopkins, 2002; Huberman, 1993; Iwanaga & Namatame, 2002). Collective means any pair of a complex system of autonomous components, together with a performance criterion by which we rank the behavior of the overall system. The performance of collective system consists of many interacting components, which we call agents, should be described on two different levels: the microscopic level, where the decisions of the individual agents occur and the macroscopic level where collective decisions can be observed. To understand the role of a link between these two levels remains one of the challenges of complex system theory.

In examining collective, we shall draw heavily on the individual decisions. It might be argued that understanding how individual make decisions is sufficient to understand and improve collective action. In this presentation, I will take a different view. Although individual decision is nested within important to understand, it is not sufficient to describe how a collection of agents arrives at specific decisions. These situations, in which an agent decision depends on the decisions of the others, are the ones that usually do not permit any simple summation or extrapolation to the aggregates. To make that connection we usually have to look at the system of interactions between agents and the collectivity.

The market entry game is a stylized representation of a common problem based on the logic of minority: a number of agents have to choose independently whether or not to undertake some activity, such as enter a market, go to a bar, or drive on a road, the utility from which is decreasing in the number of the participants. The main purpose of market entry games is to understand if and how competitive markets coordinate decentralized allocation decisions. An iterated market entry games are intended to simulate a situation where a newly emergent market opportunity may be fruitfully exploited by no more than a fixed and commonly known number of firms. The choice of market entry games for studying coordination is quite natural. When there are too many potential entrants wishing to exploit a new market opportunity, a problem arises regarding how entry may be coordinated. Without coordination, too many firms may decide to enter the market and consequently sustain losses. Conversely, fully aware of the consequences of excessive entry, firms may be reluctant to enter and exploit the market in the first place.

Market entry games, for instance, typically admit a large number of Nash equilibria. Given this multiplicity of equilibrium outcomes, an obvious question is which type of equilibrium are agents likely coordinate upon? They showed that there is no support for convergence to equilibrium play on either the aggregate or individual level or for any trend across rounds of play to maximize total group payoff by lowering the frequency of entry. The coordination failure is attributed to certain features of the payoff function that induce strong competition in the attempt to penetrate the market. They observed that under their linear payoff condition their experimental data were inconsistent with the mixed strategy equilibrium and that inexperienced subjects failed to converge to a pure-strategy equilibrium. Their analysis of the data of individual subjects clearly rejects the mixed strategy equilibrium hypothesis.

They find no support for the symmetric mixed strategy equilibrium solution on the individual level. They find no support for equilibrium play, pure or mixed, on the aggregate level. They also report that the subjects recall the last outcome and adjust their behavior accordingly, entering more frequent after a successful entry and staying out more often after a previous successful non-entry. They also suggest that coordination success or failure may depend on certain features of the payoff function and the stability of the associated equilibrium solutions.

2. Design of Complex Systems with Agent-based Simulation

Agent-based simulation involves the study of many agents and their interactions (Arthur, 1994). It is regarded as a new way of doing science through experiments. Like deduction, it starts with a set of explicit assumptions. But unlike deduction, it does not prove theorems. Instead, an agent-based model generates simulated data that can be analyzed inductively. Unlike typical induction, however, the simulated data come from a rigorously specified set of rules rather than direct measurement of the real world. When the agents use adaptive rather than optimizing strategies, deducing the consequences is often impossible, and then simulation becomes necessary. We specify how the agents interact, and then observe properties that occur at the macro level.

The connection between micro-motivation and macro-outcomes will be developed through agent-based simulation, in which a population of agents is instantiated to interact according to fixed or evolving rules of behavior. Collective phenomena to be studied include the dynamics of markets, the emergence of social norms and conventions, and daily-phenomena such as traffic jams. In examining collective decisions, we shall draw heavily on the individual adaptive decisions. Within the scope of our model, we treat models in which agents make deliberate decisions by applying rational procedures, which also guide their reasoning.

An interesting problem is then under what circumstances will a collection of agents realizes some particular stable situations, and whether they satisfy the conditions of efficiency? An agent wishes to maximize her own utility and a system designer wish to implement a decentralized algorithm for maximizing the whole utility of a group (Wardrop,1952; Wolpert, 2000). While all agents understand the outcome is inefficient, acting independently is powerless to manage this collective about what to do and also how to decide. The question of whether interacting agents self-organize efficient macroscopic orders from bottom up depends on how they interact each other. We attempt to probe deeper understanding this issue by specifying how they interact each other.

Wolpert and Tumer propose that the fundamental issue is to focus on improving our formal understanding of two closely related issues concerning collective:

1. The forward problem of how the fine-grained structure of the system underlying a collective determines its complex emergent behavior and therefore its performance.
2. The inverse problem of how to design the structure of the system underlying a collective to induce optimal performance.

Self-organization is basically the spontaneous creation of a globally coherent pattern out of the local interactions between initially independent components. This collective order is organized in function of its own maintenance, and thus tends to resist perturbations. The theory of self-organization has many potential—but as yet relatively few practical— applications. In principle, it provides an insight in the functioning of most of the complex systems that surround us. Such an understanding does not necessarily lead to a better capability of prediction, though, since the behavior of self-organizing systems is unpredictable by its very nature.

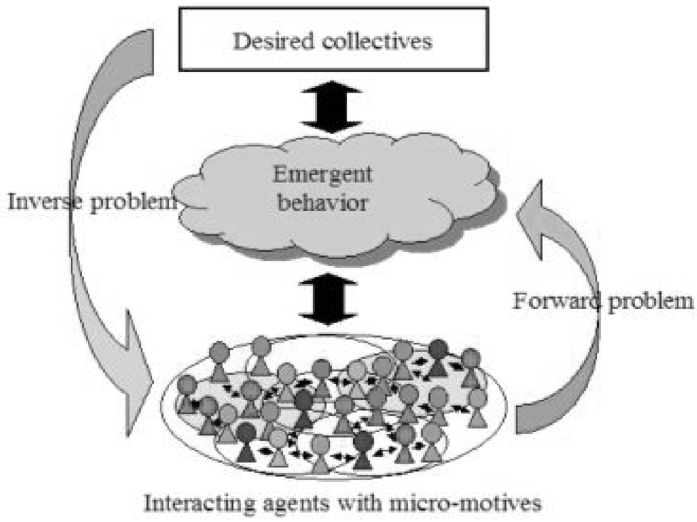

Figure 1
Solving forward and inverse problems with agent-based simulation

Most practical applications until now have focused on designing and implementing artificial self-organizing systems in order to fulfill particular functions. Such systems have several advantages over more traditional systems: robustness, flexibility, capability to function autonomously while demanding a minimum of supervision, and the spontaneous development of complex adaptations without need for detailed planning.

The basic method is to define a fitness function that distinguishes better from worse solutions, and then create a system whose components vary relative to each other in such a way as to discover configurations with higher global fitness.

For example, the collective behavior of ants, which produce a network of trails that connects their nest in the most efficient way to different food sources, has been a major source of inspiration for programs that try to minimize the load on a heavily used communication network by adapting the routes that the information packets follow to the variable demand.

Perhaps the most challenging application would be to design a complex socio-economic system that relies on self-organization rather than centralized planning and control. Although our present organizations and societies incorporate many aspects of self-organization, it is clear that they are far from optimal. Yet, our lack of understanding of social self-organization makes it dangerous to introduce radical changes, however well intended, because of their unforeseen side-effects. Better models and simulations of social systems may be very useful in this respect.

We consider dispersion problems in which an agent receives a payoff if she chooses the same action as the minority does. Agents myopically adapt their behavior based on their idiosyncratic rule to others' behaviors. We analyze adaptive dynamics that relate the collective with the underlying individual adaptive rules.

3. Collectives with Logic of Minority

There are many situations where independent agents face problems of sharing and distributing limited resources in an efficient way. For realizing efficient and equitable distributions of limited resources, agents normally react to aggregate of others' decisions. The resulting volatile collective decision is often far from being efficient. The overall performance depends crucially on the type of interaction. Interacting agents have to coordinate their decisions with others in order to improve their utility.

We can classify interdependency among agents into two types. The typical interaction is the situation where the increased effort by some agents leads the remaining agents to follow suit, which gives multiplier effects (Ochs, 1998). We can characterize this type of situation as agents behave based on the logic of majority since they receive higher payoff if they select the same strategy as the majority does. There is another type situation where agents receive payoff if they select the opposite strategy as the majority does. Let consider a competitive routing problem of networks, in which the paths from sources to destination have to be established by independent agents. For example, in the context of traffic networks, agents have to determine their route independently. In telecommunication networks, they have to decide on what fraction of their traffic to send on each link of the network. We distinguish this type of interaction where agents behave based on the logic of minority. In this paper, we investigate collectives with logic of minority, and we have to utilize different methodology. For realizing efficient and equitable distributions of limited resources, agents normally react to aggregate of others' decisions. The resulting volatile collective decision is often far from being efficient. Many agents who behave based on the logic of minority generate complex behavior of interest. A solution to this kind of a problem invokes the intervention of an authority that finds the social optimum and imposes the optimal behavior to agents. While such an optimal solution may easy to find, the implementation may be difficult to enforce in practical situations. We also face situations where the unique or few optimal solutions exist are rare, and we usually face infinite number of solutions.

3.1. Congestion Problems and Market Entry Games

There is another type of a congestion problem with the asymmetric payoff scheme in which we have to utilize different methodology. Network equilibrium models are commonly used for the prediction of traffic patterns in transportation networks that are subject to congestion. The idea of traffic equilibrium originated as early as 1924, when F.M. Knight gave a simple and intuitive description of a postulate of route choice under congested conditions. Suppose that between two points there are two highways, one of which is broad enough to accommodate without crowding all the traffic which may care to use it, but is poorly graded and surfaced; while the other is a much better road, but narrow and quite limited in capacity. If a large number of trucks operate between the two termini and are free to choose either of the two routes, they will tend to distribute themselves between the roads in such proportions that the cost per unit of transportation will be the same for every truck on both routes. As more trucks use the narrower and better road, congestion develops, until at a certain point it becomes equally profitable to use the broader but poorer highway."

In 1952 Wardrop stated two principles that formalize this notion of equilibrium and introduced the alternative behavior postulate of the minimization of the total travel costs. Wardrop's first principle of route choice, which is identical to the notion postulated by Knight, became accepted as a sound and simple behavioral principle to describe the spreading of trips over alternate routes due to congested conditions. Wardrop's first principle: "The journey times in all routes actually used are equal and less than those which would be experienced by a single vehicle on any unused route." Each user non-cooperatively seeks to minimize his cost of transportation. The traffic flows that satisfy this principle are usually referred to as "user equilibrium" (UE) flows, since each user chooses the route that is the best. Specifically, a user-optimized equilibrium is reached when no user may lower his transportation cost through unilateral action. On the other hand, the system optimal is characterized by Wardrop's second principle (SO): "At equilibrium the average journey time is minimum." This implies that each user behaves cooperatively in choosing own route to ensure the most efficient use of the whole system. Traffic flows satisfying Wardrop's second principle is generally known as the system optimal (SO) which requires that users cooperate fully or that a central authority controls the transportation system.

We can formulate network equilibrium models as the collectives with the logic of minority. Let suppose that there are two alternative route A and B to commute for a collection of agents. Each agent has the following two strategies:

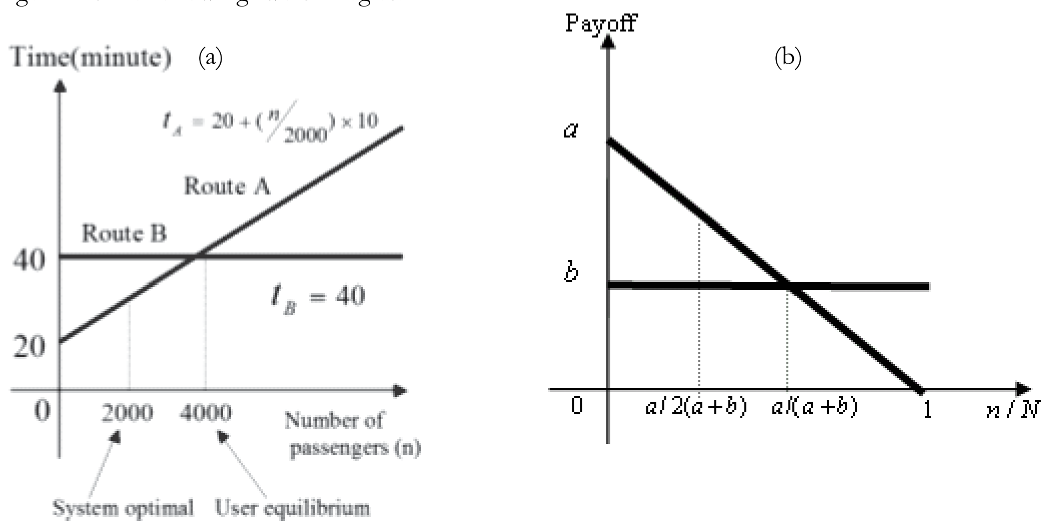

Figure 2 A Transportation Route Selection Problem
(a) Required time of two competitive routes (b) The payoff scheme of two routes

S_1(Route A): Uses a train, S_2(Route B): Uses a private car (2.1)

The required time by choosing the public transportation, a train (Route A) is constant and it is 40 minutes. On the other hand, the required time by choosing the car is the function of the number of agents who also choose the cars as shown in Figure 2a. User equilibrium is realized at the intersection in Figure 2a. However, system optimal achieved at $n = 2000$, which is the half of the user equilibrium.

If the utility of an agent is defined as the benefit minus time (cost), we can reformulate the above network equilibrium model in which each agent receives the utility when she choose distinct strategy from the majority does. We define the utility function of each agent as follows:

User equilibrium is realized at the intersection in Figure 2b, which is achieved at $n/N = a/(a+b)$, and the system optimal achieved $n/N = a/2(a+b)$, which is also the half of the user equilibrium.

3.2. The El Farol bar problem and minority games

The El Farol bar problem and its variants minority games or dispersion games have regained interests of many researchers. Similarly, we consider a competitive routing problem of networks, in which the paths from sources to destination have to be established by independent agents. For example, in the context of traffic networks, agents have to determine their route independently. In telecommunication networks, they have to decide on what fraction of their traffic to send on each link of the network. The utility of each agent is determined what the majority does, and each agent gains utility only if she chooses the opposite route of the majority does.

Let consider the following specific situation: There are two parallel routes A and B, and each agent has to choose independently one of the two routes. Each agent has the following two strategies:

S_1:Chooses the route A, S_2: Chooses the route B (2.3)

The utility of each agent is determined what the majority does, and each agent gains utility only if she chooses the opposite route of the majority does. We define the utility function of each agent as follows:

$$U(S_1) = a(1 - n/N), \qquad U(S_2) = b(n/N) \qquad (2.4)$$

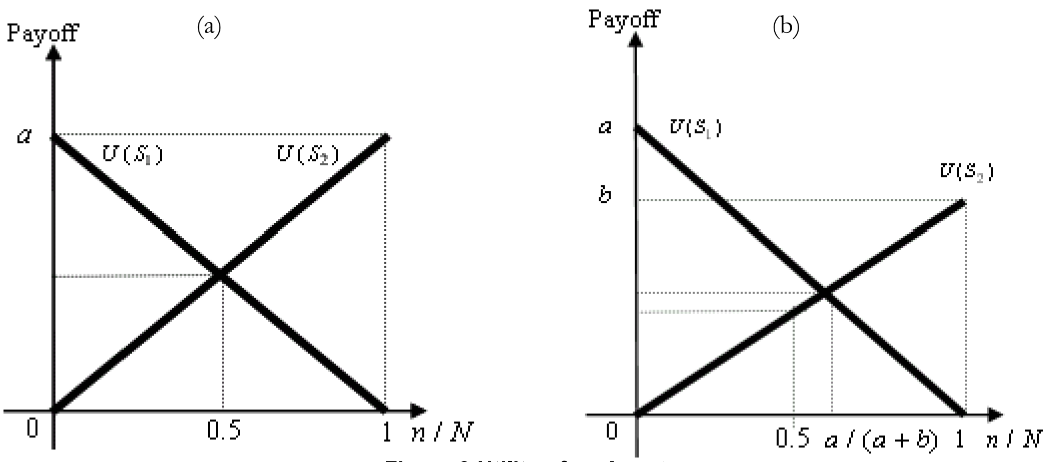

Figure 3 Utility of an Agent

Here, the utility of an agent is the function of the ratio of S_1(n/N). (a) The payoff scheme with $a = b$. (b) The payoff scheme with $a \neq b$.

The payoff of each agent under the strategy S_1 is given as a linearly decreasing function of the ratio of agents to choose the same the strategy $S_1(n/N)$ as shown in Figure 3. On the other hand, the payoff when she chooses the opposite strategy S_2 is given as a linearly increasing function of the same ratio.

Each user non-cooperatively seeks to maximize her utility, and "user equilibrium" is achieved when each user chooses the route that is the best. Specifically, a user-optimized Nash equilibrium is reached when no user may improve through unilateral action. Such Nash equilibrium is achieved at the intersection in Figure 3, and $n/N = a/(a+b)$. The collective efficiency can be measured from the average payoff of one agent. Consider the extreme case where only one agent take one action S_1, and all the other agents take the other strategy S_2. The lucky agent gets a reward a, and nothing for the others. With the payoff scheme defined in Figure 3b, each agent is more likely to prefer to choose S_1.

4. Reduction of Global Payoff Functions to Local Payoff Functions

With the view of reductionism, every phenomenon we observe can be reduced to a collection of components whose movement is governed by the deterministic laws of nature. In such reductionism, there seems to be no place for novelty or creativity. Twentieth century science has slowly come to the conclusion that such a philosophy will never allow us to explain or model the complex world that surrounds us.

Many adaptive models have been proposed such as best-response dynamics or payoff improving learning, and soon. However, it is notoriously difficult to formulate adaptive dynamics that guarantee convergence to Nash equilibrium. We call a dynamical system uncoupled if the adaptive dynamic for each agent does not depend on the payoff functions of the other agents. Hart showed that there are no uncoupled dynamics that are guaranteed to converge to Nash equilibrium. Therefore coupling among agents, that is, the adjustment of an agent's strategy depend on the payoff functions of the other agents is a basic condition for adaptive dynamics.

It is important to consider with whom an agent interacts and how each agent decides his action depending on others' actions. Agents may adapt based on the aggregate information representing the current status of the whole system (global adaptation) as shown in Figure 4a. In this case, each agent chooses an optimal decision based on aggregate information about how all other agents behaved in the past. An agent calculates her reward and plays her best response strategy. An important assumption of global adaptation is that they receive knowledge of the aggregate.

The dispersion problem with N-persons formulated with the payoff function in (3.1) or (3.2) can be decomposed into the interaction problem between an individual and the aggregate with the payoff matrix given in Table 1a or 1b.

(a) (b)

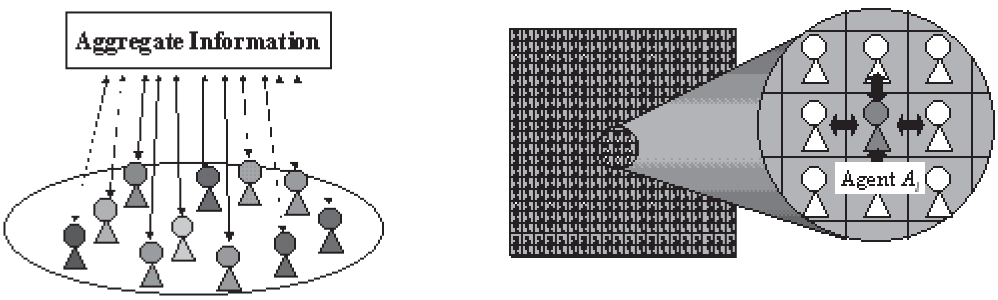

Figure 4 Reduction of Global Interaction into Local Interaction
(a) interaction between agents and collective. (b) Local interactions of agents

Table1 The Payoff Matrix of Dispersion Problems

(a)

		choice of the other	
		S_1	S_2
own choice	S_1	0 / 0	$1-\theta$ / θ
	S_2	θ / $1-\theta$	0 / 0

(b)

		choice of the other	
		S_1	S_2
own choice	S_1	0 / 0	$1-\theta$ / 1
	S_2	1 / $1-\theta$	$1-\theta$ / $1-\theta$

(a) desired collective: $n/N = 0.5$; (b) desired collective: $n/N = 1 - \theta$;
$\theta = b/(a+b)$; $0 \leq \theta \leq 1$

5. Coupling Rules of Agents and Co-evolution of Coupling Rules

An important aspect of collectives is the learning strategy adapted by individuals. We make a distinction between evolutionary systems and adaptive systems. The adaptation may be at the individual level through learning, or it may be at the population level through differential survival and reproduction of the more successful individuals. Either way, the consequences of adaptive processes are often very hard to deduce when there are many interacting agents following rules that have nonlinear effects.

Several learning rules have been found to lead an efficient outcome when agents learn from each other (Banerjee,1999, Murchland,1970). Among the adaptive mechanisms that have been discussed in the learning literature are the following (Durlauf & Young, 2002, Duffy & Hopkins, 2002, Iwanaga & Namatame, 2001).

1. *Reinforcement learning*
   Agents tend to adopt actions that yielded a higher payoff in the past, and to avoid actions that yielded a low payoff. Payoff describes choice behavior, but it is one's own past payoffs that matter, not the payoffs of the others. The basic premise is that the probability of taking an action in the present increases with the payoff that resulted from taking that action in the past (Duffy & Young, 2002).
2. *Best response learning*
   Agents adopt actions that optimize their expected payoff given what they expect others to do. In this learning model, agents choose the best replies to the empirical frequencies distribution of the previous actions of the others.
3. *Evolutionary learning*
   Agents who use high-off payoff strategies are at a productive advantage compared to agents who use low-payoff strategies, hence the latter decrease in frequency in the population over time (natural selection). In the standard model of this situation agents are viewed as being genetically coded with a strategy and selection pressure favors agents that are fitter, i.e., whose strategy yields a higher payoff against the population.

We take a different approach by focusing co-evolution. Co-evolutionary dynamics differ, in this sense, from the common use of the genetic algorithm, in which a fixed goal is used in the fitness function and where there is no interaction between individuals. In the genetic algorithm, the focus is on the final result what is the best or a good solution. In models of co-evolutionary systems, one is

(a) (b)

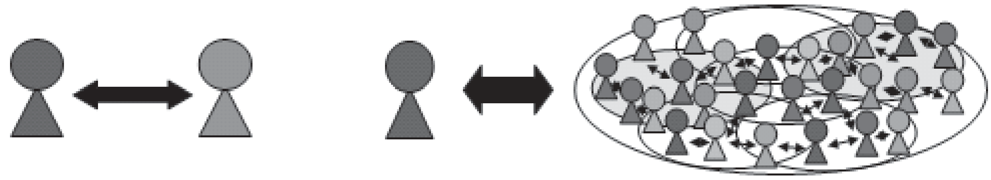

Figure 5 The Way of Coupling
(a) coupling between agents; (b) coupling between agent and collective

usually interested in the transient phenomenon of evolution, which in the case of open-ended evolution never reaches an attractor.

Each strategy in the repeated game is represented as a binary string so that the genetic operators can be applied, and we represent each strategy as $S_1 = 0$ and $S_2 = 1$. We use a memory of an agent, which means that the outcomes of the previous moves are used to make the current choice. There are 16 possible combinations of strategies between two agents if they have memory of two as shown in Table 2. We can fully describe a deterministic strategy by recording what the strategy will do in each of the 16 different situations that can arise in the iterated game.

In order to accomplish this we treat each strategy as deterministic bit strings. Since no memory exists at the start, an extra 4 bits are needed to specify a hypothetical history. At each generation, agents repeatedly play the game for T iterations. Agent i, $i \in \{1,\ldots,N\}$, uses a binary string i to choose his strategy at iteration t, $t \in \{1,\ldots,T\}$. Each position of a binary string as follows: The first position, p_1 encodes the action that agent takes at iteration $t = 1$. A position $p_j, j \in \{2,3\}$ encodes the memories that agent i takes at iteration $t - 1$ and his opponent. A position $p_j, j \in \{4,...,7\}$, encodes the action that agent i takes at iteration $t > 1$, corresponding to the position $p_j, j \in \{2,3\}$. An agent i compares the position $p_j, j \in \{2,3\}$ and decision table as shown in Table 2, and then, an agent i decide the next action. Here is an example of binary string given the agent's action taken in the previous iteration.

pattern no.	past action				next action
	$t-2$		$t-1$		
	own	opp.	own	opp.	
#1	0	0	0	1	#
#2	0	0	1	0	#
#3	0	0	1	1	#
#4	0	1	0	0	#
#5	0	1	0	1	#
#6	0	1	1	0	#
#7	0	1	1	1	#
#8	1	0	0	0	#
#9	1	0	0	1	#
#10	1	0	1	0	#
#11	1	0	1	1	#
#12	1	1	0	0	#
#13	1	1	0	1	#
#14	1	1	1	0	#
#15	1	1	1	1	#
#16	0	0	0	0	#
# represents 0 or 1					

Table 2 A Coupling Rule Between Agents

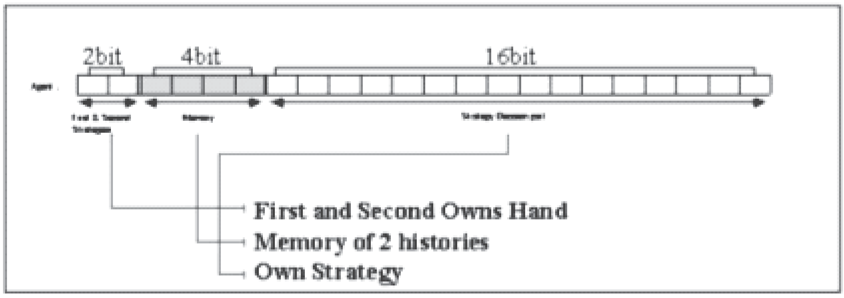

Figure 6 The Representation of Coupling Rule in Table 2

We use evolutionary learning where agents learn from the most successful neighbors, and they co-evolve their strategies over time. Each agent adapts the most successful strategy as guides for their own decision (individual learning). Hence their success depends in large part on how well they learn from their neighbors. If the neighbor is doing well, its strategy can be imitated by others. A part of the list is replaced with that of the most successful neighbor.

We show the simulation result in Figure 7a, and average payoff was gradually increased to 0.73. We also show the highest payoff and lowest payoff, and there are lucky agents who could get the maximum payoff of 1 who always belong to the minority and the unlucky agents who gets nothing who always belong to the minority side. We then consider the implementation error of the strategy. Agents choose their strategy specified by the coupling rule. However, there exists small probability of making mistake to choose. We showed the simulation results without any error and with the error rate 5% in Figure 7b. Consequently, we can conclude that evolution learning leads to a more efficient situation in the strategic environments. Significant differences were observed when agents have small chances of making mistakes. As shown Figure 6a, the highest payoff and the lowest payoff become to be close, which imply that each agent acquires the almost the same.

The advantage of agent-based simulation is that we can investigate micro-scopic rules which induce emergent behavior at the macro level. Specifically we investigate what agents have learned by evolving their coupling rules. At beginning, each agent has a different coupling rule specified by the 16 bits information. In Figure 8, we showed the acquired coupling rules by 400 agents, which are aggregated into 15 types. The numbers in brackets represent the number of agents who acquired the same type. Those 15 types of rules as shown in Figure 8 also have the commonality in bits. We show the commonality in Table 3. From Table 3, we can induce as follows: Although we specify a coupling rules between agents so that they make choices based on the choices made at the previous

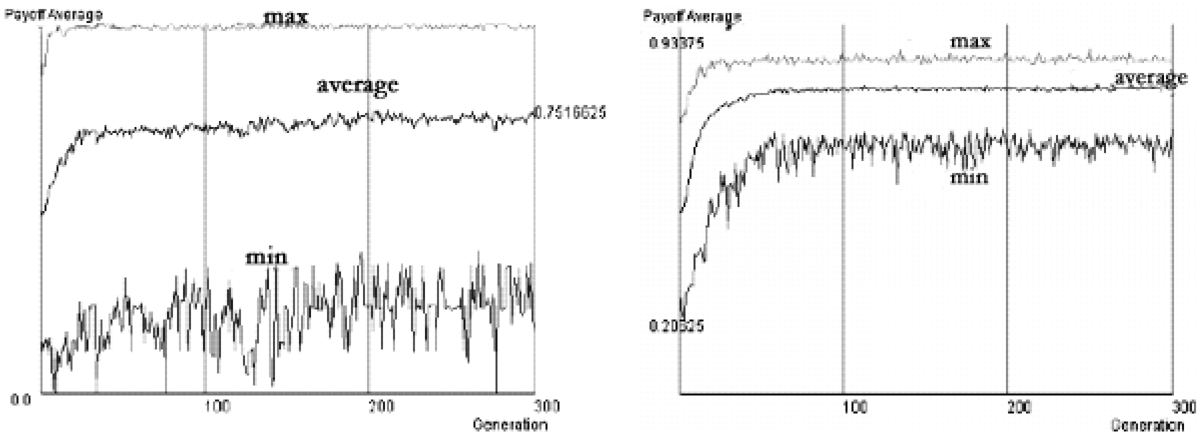

Figure 7 The Average Payoff Under Evolutionary Learning

Type01: 0 1 0 1 1 1 0 0 0 1 0 1 1 1 0 0 (10)
Type02: 0 1 0 1 1 1 0 0 0 1 0 1 0 1 0 0 (31)
Type03: 1 1 0 0 1 1 0 0 0 1 0 1 0 1 0 0 (72)
Type04: 0 1 0 0 1 1 0 0 0 1 0 1 0 1 0 0 (61)
Type05: 0 1 0 0 1 1 0 0 0 1 0 1 1 1 0 0 (10)
Type06: 1 1 0 0 1 1 0 0 0 1 0 1 1 1 0 0 (13)
Type07: 0 1 0 0 1 0 0 0 0 1 0 1 0 1 0 0 (40)
Type08: 1 1 0 1 1 1 0 0 0 1 0 1 0 1 0 0 (37)
Type09: 1 1 0 0 1 0 0 0 0 1 0 1 0 1 0 0 (43)
Type10: 0 1 0 1 1 0 0 0 0 1 0 1 0 1 0 0 (31)
Type11: 1 1 0 1 1 0 0 0 0 1 0 1 0 1 0 0 (35)
Type12: 0 1 0 0 1 0 0 0 0 1 0 1 1 1 0 0 (2)
Type13: 1 1 0 1 1 1 0 0 0 1 0 1 1 1 0 0 (12)
Type14: 0 1 0 1 1 0 0 0 0 1 0 1 1 1 0 0 (1)
Type15: 1 1 0 0 1 1 0 0 0 1 1 1 0 1 0 0 (2)

Figure 8 Types of Meta-rules Acquired by 400 Agents
The number in bracket represents the number of agents who acquired the same types of the meta-rules.

two periods. However, each agent learnt to make choice based on the previous choices. And if agents choose $S_1(0)$ and their opponent chooses $S_2(1)$ at the previous time period, then they choose $S_2(1)$. If agents choose $S_2(1)$ and their opponent chooses $S_2(0)$ at the previous time period, then they choose $S_2(0)$. These acquired rules are also interpreted as follows: if they gain then they change their strategy, and we call this rule is based on the principle of give-and-take.

From this result we can conclude that some mistakes help agents to acquire the give-and-take strategy in the long run, which lead a society of agents to be both efficient and equitable. For self-organizing systems, fitter usually means better or with more potential for growth. However, the dynamics implied by a fitness landscape does not in general lead to the overall fittest state: the system has no choice but to follow the path of steepest descent. This path will in general end in a *local* minimum of the potential, not in the *global* minimum. Apart from changing the fitness function, the

	past actions				
bit	t-2		t-1		next
position	own	opp	own	opp	action
#2	0	0	0	1	1
#3	0	0	1	0	0
#5	0	1	0	0	1
#6	0	1	0	1	0 or 1
#7	0	1	1	0	0
#8.	0	1	1	1	0
#9	1	0	0	0	0
#10	1	0	0	1	1
#11	1	0	1	0	0
#12	1	0	1	1	1
#14	1	1	0	1	1
#15	1	0	1	0	0
#16	1	1	1	1	0

Table 3 Learnt Coupling Rules Characterization as Give-and-Take

only way to get the system out of a local minimum is to add a degree of indeterminism to the dynamics, that is, to give the system the possibility to make transitions to states other than the locally most fittest one. This can be seen as the injection of noise or random perturbations into the system, which makes it deviate from its preferred trajectory. Physically, this is usually the effect of outside perturbations. Such perturbations can push the system upwards, towards a higher potential. This may be sufficient to let the system escape from a local minimum. In general, the deeper the valley, the more difficult it will be for a perturbation to make a system leave that valley. Therefore, noise will in general make the system move out of the more shallow, and into the deeper valleys. Thus, noise will in general increase fitness. The stronger the noise the more the system will be able to escape the relatively shallow valleys, and thus reach a potentially deeper valley. However, a system with noise will never be able to really settle down in a local or global minimum, since whatever level of fitness it reaches it will still be perturbed and pushed into less fit states. The most effective use of noise to maximize self-organization is to start with large amounts of noise which are then gradually decreased, until the noise disappears completely. The initially large perturbations will allow it to escape all local minima, while the gradual reduction will allow it to settle down in what is hopefully the deepest valley. This is the principle underlying annealing, the hardening of metals by gradually reducing the temperature, thus allowing the metal molecules to settle in the most stable crystalline configuration. The same technique applied to computer models of self-organization is called *simulated annealing*.

6. Synthesis of Agents Behaving with Coupling Rules

The give-and-take rule departs from the conventional assumption such that agents update their behaviors in order to improve their measure functions such as payoffs. It is commonly assumed that agents tend to adopt actions that yield a higher payoff in the past, and to avoid actions that yield a low payoff. With the give-and-take, on the contrary, agents are assumed that they yield to others if they receive the payoff by taking the opposite strategy at the next time period, and they choose randomly if they do not gain the payoff. Then we will formalize the payoff scheme with give-and-take strategy.

We denote the status of the collective choice by the state variable $\omega(t)$ as follows:

1) $\omega(t) = 0$ if $A(t) < N\theta$ (less crowded) (6.1)

2) $\omega(t) = 1$ if $A(t) \geq N\theta$ (over crowded) (6.2)

In the case when the attendance is below than the capacity, the number of agents to stay at home is greater than the capacity. We also consider a population of agents with $N = 2500$ with the capacity rate $\theta = 0.5$. Figure 9 shows the simulation result when all agents adapt the give-and take learning rules in (6.2). Figure 9a shows the number of agents having chosen S_1 and S_2 over time, and it is shown that the average number of agents who choose S_1 (Go) converges to the capacity, indicating that collective behavior satisfies the constraint. Figure 9b shows the proportion of agents with the same average payoff. The majority of agents receive the average payoff 0.5. This result indicates that not only social efficiency, but also social equality is achieved with give-and-take strategy.

As shown in Section 3, the average payoff per agent is given by $2\theta(1 - \theta)$ which takes the minimum value at the capacity rate $\theta = 0.5$. We evaluate the performance of give-and take learning with the capacity rate $\theta = 0.6$, the capacity rate of the El Farol problem. In Figure 10, we showed the simulation result. The Figure 10a shows the number of agents of having chosen S_1 and S_2 over time. It is shown that the average number of agents who choose S_1 (Go) converges to the capacity given by Nθ. In Figure 10b, we showed the proportion of agents with the same average payoff. The majority of agents received the payoff less than 0.5, which is less than the average payoff at the efficient collective behavior.

This result indicates that the problems of inefficiency and inequity become to be crucial if the capacity rate θ deviates from 0.5, and increasing the asymmetry of the minority and majority sides.

This implies that we may need the central authority in order to achieve both social efficiency and equity in asymmetric situations.

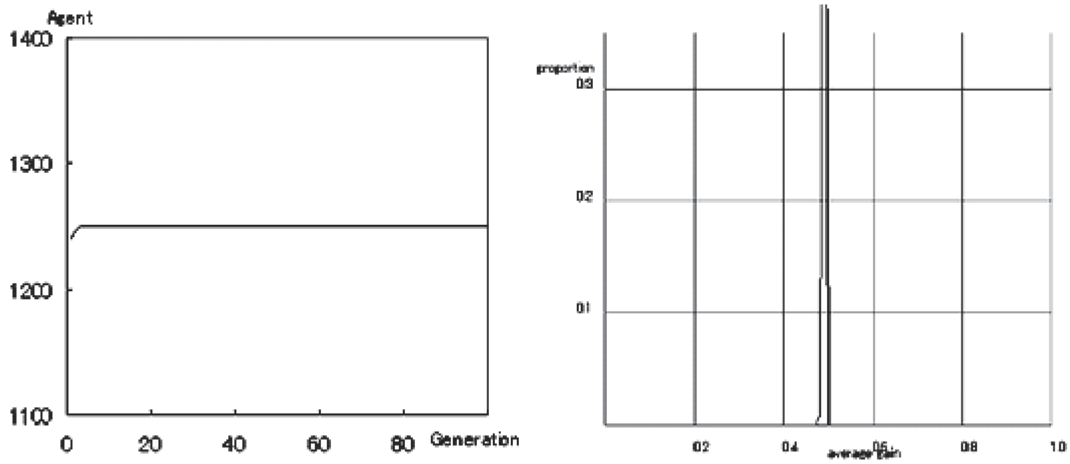

Figure 9 The Simulation Result of
Interacting Agents with the Rule of Give-and-Take (θ = 0.5)
(a) the dynamic changing of numbers of agents of having chosen S_1 and S_2(b) the proportion of agents with the same payoff

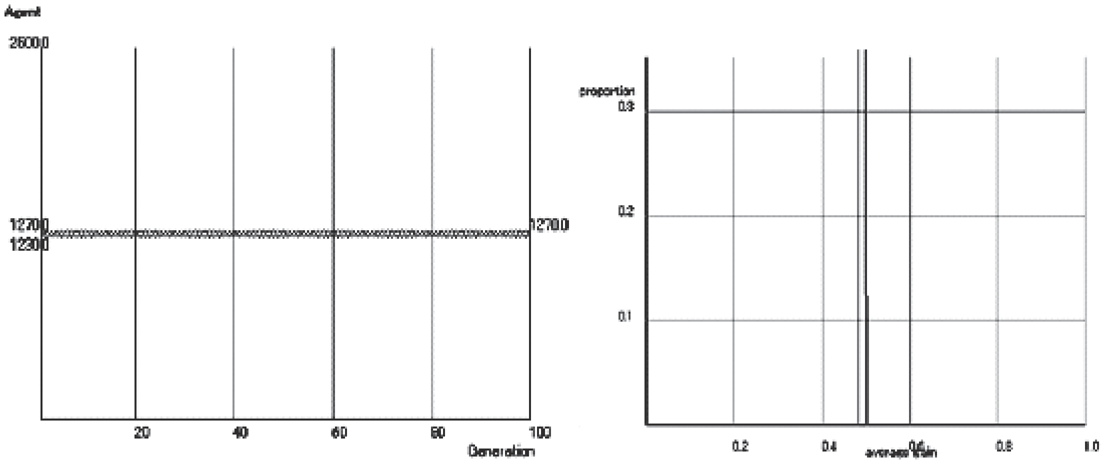

Figure 10 The Simulation Result of
Interacting Agents with the Rule of Give-and-Take (θ = 0.6)
(a) the dynamic changing of numbers of agents of having chosen S_1 and S_2; (b) the proportion of agents with the same payoff

7. Exogenously Design of Individual Payoff Functions

Many researchers have pointed out that equilibrium theory does not resolve the question of how groups of agents behave in a particular interdependent decision situation. They argued that "It is hard to see what can advance the discussion short of assembling a collection of people, putting them in the situation of interest, and observing what they do" Faced with this challenge, experimenters have devised a large variety of games with multiple pure-strategy equilibria coordination games investigated to study coordination with no preplay communication in the controlled environment of the laboratory.

Included in these games are market entry games whose purpose is to understand if and how competitive markets coordinate decentralized allocation decisions. An iterated market entry games are intended to simulate a situation where a newly emergent market opportunity may be fruitfully exploited by no more than a fixed and commonly known number of firms. The choice of market entry games for studying coordination is quite natural. When there are too many potential entrants wishing to exploit a new market opportunity, a problem arises regarding how entry may be coordinated. Without coordination, too many firms may decide to enter the market and consequently sustain losses. Conversely, fully aware of the consequences of excessive entry, firms may be reluctant to enter and exploit the market in the first place.

Rapoport investigated coordination behavior in yet another class of market entry games which include only a single market with a commonly known capacity This game is played by a group N of n symmetric agents. On each period, an integer c such that $1 < c < n$, interpreted as the "capacity of the market", is publicly announced. Then each agent $i \in N$ must decide privately whether to enter the market ($x_i = 1$) or stay out of it ($x_i = 0$). The payoff for each stage game is determined from:

$$U_i(\mathbf{x}) = \begin{cases} v & \text{if} \quad x_i = 0 \\ v + r(c - m) & \text{if} \quad x_i = 1 \end{cases} \tag{7.1}$$

where $U_i(\mathbf{x})$ is agent i's payoff, and $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ is the vector of individual decisions, m is the number of entrants ($0 < m < n$); and v, r, and c are positive constants.

The constant c represents the capacity of the market (or road or bar). This market entry game has complete information, binary actions, and an incentive to enter the market which decreases linearly in the number of entrants. It allows each agent the option of staying out of the market and receiving a positive or negative payoff v, which does not depend on the decisions of the of the other n - 1 agents.

Market entry games typically admit a large number of Nash equilibria. Pure equilibria involve coordination on asymmetric outcomes where some agents enter and some stay out. The only symmetric outcome is mixed, requiring randomization over the entry decision. There also exist asymmetric mixed equilibria, where some play pure while others randomize. Given this multiplicity of equilibrium outcomes, an obvious question is which type of equilibrium are agents likely coordinate upon? They showed that there is no support for convergence to equilibrium play on either the aggregate or individual level or for any trend across rounds of play to maximize total group payoff by lowering the frequency of entry. The coordination failure is attributed to certain features of the payoff function that induce strong competition in the attempt to penetrate the market. They observed that under their linear payoff condition their experimental data were inconsistent with the mixed strategy equilibrium and that inexperienced subjects failed toconverge to a pure-strategy equilibrium. Their analysis of the data of individual subjects clearly rejects the mixed strategy equilibrium hypothesis.

An another observation conclusion concerns a shift in emphasis in the research on coordination in iterated market entry games. Clearly, there is no compelling reason to believe, *a priori*, that agents sharing no common history on which to condition their choices will achieve perfect coordination in these games. Nor is there any particular reason to believe that agents can randomize their choices across iterations of the same stage game, producing sequences of responses that pass statistical tests for independent and identically distributed events.

They find no support for the symmetric mixed strategy equilibrium solution on the individual level. They find no support for equilibrium play, pure or mixed, on the aggregate level. They also report that the subjects recall the last outcome and adjust their behavior accordingly, entering more frequent after a successful entry and staying out more often after a previous successful non-entry. They also suggest that coordination success or failure may depend on certain features of the payoff function and the stability of the associated equilibrium solutions.

Previous experimental studied have yielded that Nash equilibrim fails as a predictor of human behavior. Duffy and Hopkins studied the long-run predictions of learning models in the context of the market entry games. They investigated the hypothesis that individual should learn equilibrium behavior. Agents are modeled to learn with reinforcement learning, and the probability of entering or staying out is reinforced with performance of the previous play. They showed a collection of agents converge to asymmetric pure equilibrium that involves what they call "sorting" where some agents always enter and the remaining agents always stay out.

Similar observation has been also reported by Bell (Duffy & Hopkins, 2002). She studied the long-run predictions of learning models in the context of the minority games. Agents are modeled to learn with reinforcement learning. She showed a collection of agents converge to asymmetric pure equilibrium that involves what she call "sorting" where some agents always enter and the remaining agents always stay out.

8. Summary: Evolutionary Design of Complex Systems with Agent-based Simulation

Many fundamental changes to a society result from collective behaviors of groups of interacting agents. How do heterogeneous micro-world of individuals generate the global macroscopic orders and regularities of society? Much of hidden knowledge is underlying accumulated interactions. There is no presumption that a collection of interacting agents leads to collectively satisfactory results without any central authority. The system performance of interacting agents crucially depends on the type of interactions among agents as well as how they adapt to others. There are two closely related issues concerning collective, (1) the forward problem of how the fine-grained structure of the system underlying a collective determines its complex emergent behavior and therefore its performance, and (2) the inverse problem of how to design the structure of the system underlying a collective to induce optimal performance. We discuss how agent-based simulation contributes to solve these two issues.

In this paper we addressed the issue of collective decisions by agents in which they have to realize both efficient and equitable utilization of limited resources. Agents normally react to aggregate of others' decisions, and the resulting volatile collective decision is often far from being efficient. By means of experiments, we showed that the overall performance depends crucially on the types of interacting decisions as well as the heterogeneity of agents in term of their preferences. We considered two different types of interaction formulated as the coordination problem and the dispersion problem. We showed that the most crucial factor that considerably improves the performance of the system of interacting agents is the endogenous selection of the partners and reinforcement of preferences at individual levels.

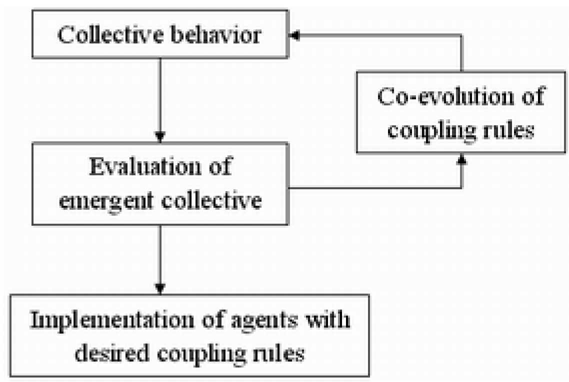

Figure 10 An Evolutionary Design with Agent-based Simulation

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