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Attractor of Logic
On dualist liar's paradox with fuzzy logic

YUN HARIADI
Dept. Dynamical System Modeling, Bandung Fe Institute
Jl Cemara 63 Bandung, Indonesia
yh@dynsys.bandungfe.net

Abstract
This paper is an extended version on liar's paradox with fuzzy logic. The emergence of logic attractor is observed in bifurcation diagram with parameter control as truth-value on fuzzy logic. The simulation employs two cases, in which the second case is the qualitative extremeness or very-ness of the first case. The simulation result shows that in the first case the interval of attractor is increasing as the control parameter increases, while in the second case, the coordinate of attractor changes from two points of attractor (order) into unlimited points of attractor (chaotic) at control parameter 0.25. This change of logic of attractor number shows inconsistency on liar paradox.

Keywords: dualist liar's paradox, fuzzy logic, logic attractor, chaos, inconsistency

1. Introduction

Paradox is the fundamental weakness in the basic of binary logic system, which in the future turns to the born of Gödel incompleteness theorem. Far before Christian years, Zeno had pointed out his doubt on logic, which then known as Zeno's paradox. In its development, the paradox problem continues. It was triggered by the rise of definition of set by Cantor, which later known as Naïve Set Theory.

One of the unique characteristics of this paradox is the *self-reference*, although the presence of this self-reference is not necessary and sufficient condition to the presence of paradox. Nevertheless, greater attention on self-reference types arose, since most of paradox considers self-reference. Bolander (2002) has deeply studied the influence of self-reference to paradox and further come up with several conditions to the occurrence of paradox by the presence of self-reference. Furthermore, the self-reference is used in the basic model of Artificial Intelligent to describe the introspective agent. An introspective agent is an agent capable to retrospect herself on her thought, belief, and plan (Bolander, 2002). One of the important and unique characters of the introspective agent is the presence of *believe* operator, that is a condition for agent to believe a sentence in a formal theory *K*.

Tarski through his *schema* method (known as Schema T) built a tool for deciding a paradox in a sentence. He introduced operator $\ulcorner . \urcorner$ as an injective function over a statement (sentence) to a number, which is strengthened by Tarski's theorem that, "Peano Arithmetic extended with schema T is inconsistent". The schema applied on introspective agent is showing paradox that finally produces inconsistency on the agent itself.

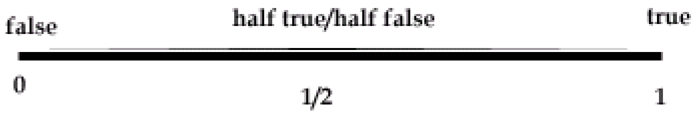

Figure 1 Fuzzy Logic

In fuzzy logic, we are allowed to say that a statement is both half-true and half-false.

In the meantime, logic has grown up on the truly new direction with the birth of fuzzy logic. Lofti Zadeh has extended fuzzy set as basic formalism to fuzzy logic. Fuzzy logic gives new hope to solve paradox in the binary logic. Different from binary logic based on binary truth-value: true (1) and false (0), in fuzzy logic there is many-valued truth in the interval [0,1].

How can fuzzy logic give a new hope for solving paradox on binary logic such that in case of paradox, there lies inconsistencies? For simple example we have

"This sentence is false" (1)

If the statement has truth-value p, then it also has truth-value $q = 1 - p$. The sentence is consistent if $p = q$ or $p = 1 - p$, such that in the binary logic, the value of p and q is either 1or 0. If $p = 1$ then $q = 1 - 1 = 0, p \neq q$, and there is a contradiction. On the contrary, if $p = 0$ then $q = 1 - 0 = 1, p \neq q$, and there is still a contradiction. Simply put, to say that liar paradox has both true and false values in the binary logic, is to say that the paradox has, as what we have known, inconsistency in binary logic.

However, for a truth-value in interval [0,1] for many-valued logic, $p = 1 - p$ will have solution $p = ½$, such that it only happens in fuzzy logic: the truth-value for the paradox above is *half true* or *half false*.

There have been wide applications of fuzzy logic in social sciences, especially on social complexity. Vladimir Dimitrov (1997) uses fuzzy logic to help describing, analyzing, understanding and eventually dealing with the paradoxical and chaotic dynamics of social systems. However, using fuzzy logic on liar paradox cannot prevent the presence of paradox we find in binary logic (Hajek, 2000). This also shows that liar paradox can be formalized in fuzzy logic (Mehta, 2000).

In other parts, dynamical system has made a great development with the emergence of chaos theory. In the chaos theory, every phenomenon has character of non-linearity. One unique characteristic of chaos theory is sensitive on initial conditions and the presence of *emergence*. The most interesting example for the emergence for chaos theory is the *attractor*. The *attractor* pulls every points surrounding it closer to its positions, is ergodic, and fill up all over phase space. On chaos theory, it is interesting to observe that order condition changes into disorder, which is shown by the change from limited attractors to unlimited attractors. Chaos theory has been widely used in social sciences such as Lotka-Volterra model for describing the relationship between the capitalists and labors by Godwin (1967), and crime attractor as an emergent factor on two way causality of crime and unemployment (Hariadi, 2003).

This paper studies liar's paradox, fuzzy logic, and chaos theory. Through a simple simulation by means of plotting a truth-value onto bifurcation diagram with control parameter as the truth-value, dualist liar paradox can be combined with fuzzy logic. The end of paper will discuss the using of this type of liar's paradox on introspective agent.

2. Model

One of the models introspective agents use is self-reference. This simulation will develop self-reference of liar paradox type. Ian Stewart (1999) has developed another type of liar paradox (the chaotic dualist):

X = "This sentence is as true as Y" (2a)
Y = "This sentence is as true as X false" (2b);

whilst for fuzzy dualist

X = "It is very true that this sentence is as true as Y" (3a)
Y = "It is very true that this sentence as true as X false" (3b).

In that paradox, Ian Stewart only showed the emergence of logical attractor through iteration of X sentence into Y sentence. This paper will further explain the model.

First, the paradox will be extended into general paradox on truth-value for each statement as follows:

X = "This sentence is alpha as true as Y" (4a)
Y = "This sentence is as true as X false" (4b);

and for general fuzzy dualist

X = "It is very true that this sentence is alpha as true as Y" (5a)
Y = "It is very true that this sentence as true as X false" (5b).

α is a truth-value for sentence X in interval [0,1]. Stewart so far has studied the truth-value of 0.5 (half). Furthermore, the generalization of this form would bring us to bifurcation diagram with control parameter for all value of α.

Second, the liar's paradox will be more developed in the simulation for introspective agent as an extended form of doxastic system. In this simulation agent with belief X would change as soon as there is new information Y. This will be discussed further at the end of paper.

The truth-value for the sentence:

P if and only if Q

is $1 - |p - q|$, with p is the truth-value for P and q is the truth-value for Q.

Let the truth value for X sentence is x and the truth value for Y sentence is y. Hence the truth-value for sentence 2a) is

$$x = 1 - |y - x|.$$

The use of this truth-value for *as true as* is similar with the usual *if and only if*. Therefore, for sentence 2b):

$$y = 1 - |(1 - x) - y|$$

with 1-x is 'falsehood'-value for sentence X and the truth-value for *very-ness* sentence is defined to be the square (quadrate) of the value of the original sentence. Suppose that the truth-value for X sentence is x such that the truth-value for "X sentence is very true" is x^2.

Thus, the truth-values for sentence 3) are:

$$x = (1 - |y - x|)^2$$
$$y = (1 - |(1 - x) - y|)^{1/2}$$

whilst for sentence 4) with truth degree represented with α, the truth-value becomes

$$x = 1 - |\alpha y - x|$$
$$y = 1 - |(1 - x) - y|$$

with α is in the interval [0.1]. Hence for sentence 5):

$$x = (1 - |\alpha y - x|)^2$$
$$y = (1 - |(1 - x) - y|)^{1/2}$$

3. Simulations and Analysis

Let us suppose that agent X has a truth-value x_i at time i and the truth-value for information Y at time i is y_i. Thus the truth-value for sentence 4) and 5) are:

$$x_{i+1} = 1 - |\alpha y_i - x_i|$$
$$y_{i+1} = 1 - |(1 - x_i) - y_i|$$

and

$$x_{i+1} = (1 - |\alpha y_i - x_i|)^2$$
$$y_{i+1} = (1 - |(1 - x_i) - y_i|)^{1/2}$$

The simulation is performed for both truth-values above. For each case we will see how the truth-value of sentence X at all value of α. The simulation runs 1000 iterations for each α and the last 200 iterations will be plotted. This method is similar with bifurcation diagram for logistic equation, as shown in Figure 2.

First case, we have initial value $x_0 = 0.6$ and $y_0 = 0.7$.

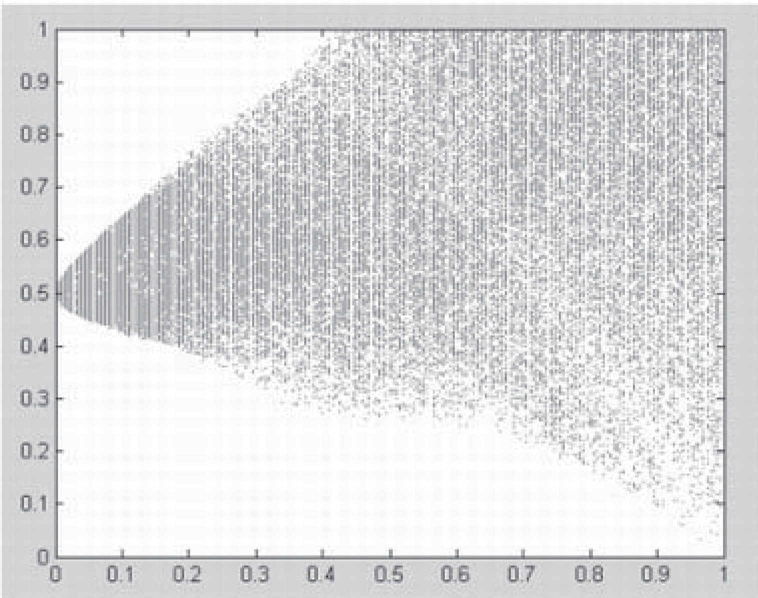

Figure 2
The larger the value α, the wider the attractor points are.

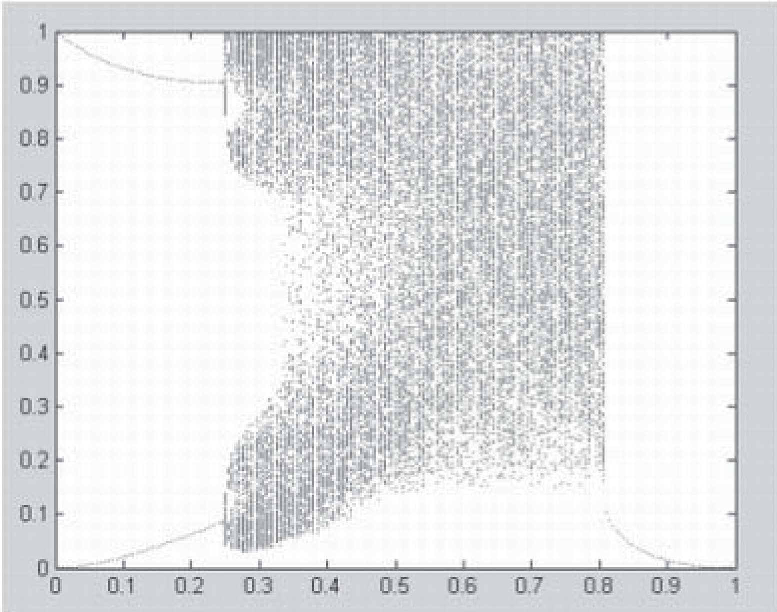

Figure 3

In initial condition, there are two attractors and then it changes drastically into infinite attractors and return to the initial condition when α near 1.

Attractors points show where truth-values of X sentence (x_i) are concentrated as shown by the increase of the truth-value α that would make the attractor dots more widespread. It is clearly seen from the truth-value that is initially concentrated to 0.5, and finally getting widespread throughout the interval [0,1]. It is interesting that attractor points tend to concentrate on the truth-value approaching 1 (at α in the interval of [0.45,1]), meanwhile in the interval, the truth-value approaching 0 will not be reached at interval α.

Second case, we have initial value $x_0 = 0.6$ and $y_0 = 0.7$.

The second case shows an interesting behavior that is similar with bifurcation behavior. At the initial condition there are two attractors and then it changes drastically into infinite numbers of attractors on α near 0.25, which indicates a chaotic region. In the beginning, attractor points have opposite truth-value such that one approaches the truth-value of 1 (true) and the other the truth-value of 0 (false). This condition is comparable to liar's paradox on binary logic that changes from 1 to 0, then back to 1 again, and so forth.

At α near 0.25, both attractors turn into chaos, and this change still emerges certain pattern, which is concentrated in the two previous attractors. This condition prevails until α approaches 0.5. This fact gives information that in the second case, the truth-value approaching 0.5 (as true as sentence Y) causes sentence X to fluctuate more smoothly to approach truth-value 1, and never reaches below 0.1.

It is also an interesting fact that, as α touches 0.8, the infinite attractor points drastically change into single attractor. This designates the change from chaotic region to order. It also informs that at truth-value α in the interval [0.8,1], it exhibits a serious consequence to sentence X such that from the truth-value of [0.12,1], it is gradually reaching 0 (false).

If we compare both cases, we can see that the presence of *very-ness* gives important influence in the formation of different patterns of attractor in each case. In the first case, the formed attractor points do not clearly show the changes of the number of attractor points. However, in this case we can clearly observe that attractors spread wider in [0,1]. In the second case, it is interesting that the number of attractor points drastically changes from two attractor points (order) to infinite attractors (chaos) and then back to single attractor (order) again.

From both cases we can understand that the presence of *very-ness* leads the truth-value to a quadratic change. Certainly this corresponds to quadratic equation on the range [0,1] that leads points near 0 to be attracted to 0 and points near 1 to 1.

4. Discussions and conclusion

The bifurcation diagram, which is performing the attractor points, can be understood as inconsistency of a truth-value of sentence *X*. This inconsistency occurs when the truth-value of sentence *X* changes as control parameter changes. In another words, a change on control parameter α will also lead to the change of the range of truth-value of sentence *X*, compared to its initial condition.

The role of *very-ness* contributes an important consequence to the forming of unique attractor points started from two attractor points to infinite attractor points and back to single attractor point. The chaotic condition is indicated by the infinite numbers of attractors. This is a direct consequence of the definition of the truth-value with *very-ness*, that quadrates the initial values. The attractor number phenomenon shows inconsistency of truth-value over sentrence *X*.

In introspective agent, an agent has capability to self-reflection, in which the action of agent as in Bolander's model (Bolander, 2000) is represented as action of "believing" a sentence. Belief operator has been developed on doxastic logic by Jaakko Hintikka, yet its development could not accommodate its capability to revise its belief.

Furthermore, in order to be more realistic in modeling behavior of agent, we should include the capability of agent to revise his belief based on new information. Doxastic logic system has been developed to cover the problem, known as AGM-approach (Alchourron, Gardenfors and Makinsaon). In what way can it happen? According to AGM-approach there are three basic types for doxastic action (Lindstrom, *et. al.*, 1999):

1. Expansion, $G + \gamma$
2. Contraction, $G - \gamma$
3. Revision, $G * \gamma$

where *G* is the set of old beliefs, and *g* the new beliefs (from new informations) that give influence *to* *G*. In the liar's paradox used in this simulation, the sentence *X* as old belief is used by agent to make decision, meanwhile sentence *X* is dependent on the truth of sentence *Y*. In other words, sentence *Y* is the new information that is able to change belief *X*. In short, it can be written as *X* in *G* and *Y* in *g*.

In the further works, a proof of liar's paradox type can be obtained with formal system through simulation implemented in schema-T.

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