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Market Share in Duopoly Game

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Abstract
This paper is based on the assumption that efforts which include innovation, cost on production and efficiency, would affect market share in duopoly game. We develop a model and simulation for two combinations of how firms decide their capacity of production based on bounded rationality and adaptive expectations which in turn lead to a pair of firm's strategies: homogeneous and heterogeneous. If some effort on market share only occur in one firm, the simulation using bifurcation map with rate of market share as control parameter showing that for the firm with homogeneous strategy, its bifurcation map will show chaotic condition at initial and this chaos will dominate most of all value control parameter. What happens on firm with heterogeneous strategy is otherwise. An interesting result is that competition which starts from homogeneous strategy in a duopoly market will end up in a monopoly for certain value of control parameter. Attractor on chaotic condition for both firms shows domination of one firm over its rival.
Keywords: market share, duopoly, monopoly, bounded rationality, adaptive expectations, bifurcation, chaos, attractor

1. Introduction

In economy, only a few firms are involved in producing certain goods. This is the case for products that require a big capital and advanced technology. This includes military industry, computer industry, and aerospace industry.

In competition model, at least two firms are required. One of the simplest forms is duopoly model, which only involves two firms. It has been a long time known to be interesting since early model of Cournot for its character of chaos. Modification on this model focuses on the strategy that the firms use: homogeneous and heterogeneous; and the expectation of the output the firms have to maximize: bounded rationality and adaptive expectation (Agiza & Elsadany, 2004). In this model of duopoly game, profit depends on production output and marginal profit as expected over its rival.

What if one firm applies some effort according to current market share? How can this affect production output for both firms and how can the rate of market share influence over output production for both firms? The structure of this paper is as follows: Section 2 is about the duopoly model originated from Agiza that is modified with market share factor; Section 3 performs numerical simulation followed by analysis on bifurcation map and also attractor on chaos control parameter; all concluded in Section 4 along with direction it gives for future works.

2. Model

In our duopoly game, the production of goods of the two firms *j* and firm *-j* is complement. The main problem in duopoly game is to determine output production for the future. We build model for market share on duopoly game over two different cases: homogeneous strategy and heterogeneous strategy. These strategies are combination of bounded rationality and adaptive expectation. In advance, this paper modifies these two kinds of strategies with the presence of some effort related to market share.

Agiza & Elsadany (2004) have developed Cournot model on simple duopoly game. Given $q_j(t)$ the production output of firm *j* at time *t*, then for the production output of firm *j* and its rival *-j* is:

$$q_j(t+1) = \arg\max_{q_j} \Pi_j\left(q_j(t), q^e_{-j}(t+1)\right); j = i, -i \qquad \text{(i)}$$

with Π_j is profit function of firm *j*, and $q^e_{-j}(t+1)$ is expectation of firm *j* over firm *-j* at time $t+1$ on constant price. The model in i) is developed for players with bounded rationality. The boundedness comes from the limited information about the market and the rival. The firm will decide its production output at *t*+1 based on production output at *t* and marginal profit. The equation becomes:

$$q_j(t+1) = q_j(t) + \alpha_j q_j(t)\frac{\partial \Pi_j}{\partial q_j} \qquad \text{(ii)}$$

Furthermore, the model in ii) will be modified by applying some effort related to market share factor, based on the assumption that the effort by the firm *j* will increase output productions and will decrease output productions on the firm *-j*. We can simply write this as:

$$\overline{q}_j = q_j + \beta q_{-j} \qquad \text{(iiia)}$$

$$\overline{q}_{-j} = q_{-j} - \beta q_{-j} \qquad \text{(iiib)}$$

with β is the rate of market share over production output. Hence ii) becomes:

$$q_j(t+1) = \overline{q}_j(t) + \alpha_j \overline{q}_j(t)\frac{\partial \overline{\Pi}_j}{\partial \overline{q}_j} \qquad \text{(iv)}$$

Suppose we have demand function of the form:

$$P = a - bQ \tag{v}$$

with $Q = q_j + q_{-j} = \overline{q}_j + \overline{q}_{-j}$ and production cost function and cost of the effort on market share are respectively:

$$\overline{C}_j(q_j) = \overline{c}_j q_j \tag{via}$$

$$\overline{c}_j = c_j + \gamma\beta \tag{vib}$$

With pure production cost (i.e.: no cost for manipulating market share whatsoever) we have $C_j(q_j) = c_j q_j$ such that:

$$\gamma = \begin{cases} 1 & \text{effort on market share} \\ 0 & \text{otherwise} \end{cases}$$

The profit function hence becomes:

$$\overline{\Pi}_j(\overline{q}_j, \overline{q}_{-j}) = \overline{q}_j(a - bQ) - \overline{c}_j \overline{q}_j \tag{vii}$$

and

$$\frac{\partial \overline{\Pi}_j}{\partial \overline{q}_j} = a - \overline{c}_j - 2b\overline{q}_j - b\overline{q}_{-j} \tag{viii}$$

We can obtain the optimum function of the form:

$$\overline{q}_j = \frac{1}{2b}(a - \overline{c}_j - b\overline{q}_{-j}) \tag{ix}$$

Substituting ix) to iv):

$$q_j(t+1) = \overline{q}_j(t) + \alpha_j \overline{q}_j(t)(a - \overline{c}_j - 2b\overline{q}_j(t) - b\overline{q}_{-j}(t)); j = i, -i \tag{x}$$

The equation above shows the change of production output at t+1 based on marginal profit and output production at t. Since both firms use the same kid of bounded rationality, then the strategy is known as homogenous.

Assume the firm j in its decision making process uses production output and its rival's reaction function $g(q_{-j})$. This strategy is known as adaptive expectation, simply written as:

$$q_j(t+1) = (1-\nu)q_j(t) + \nu g(q_{-j}(t)) \tag{xi}$$

With some effort on market share factor, xi) becomes:

$$q_j(t+1) = (1-\nu)\bar{q}_j(t) + \nu g(\bar{q}_{-j}(t)) \tag{xii}$$

Substituting optimal function ix) to :

$$q_j(t+1) = (1-\nu)\bar{q}_j + \frac{\nu}{2b}(a - \bar{c}_{-\ j} - b\bar{q}_j) \tag{xiii}$$

Therefore, for both firms with different strategies, we have:

$$q_1(t+1) = \bar{q}_1(t) + \alpha\bar{q}_1(t)(a - \bar{c}_1 - 2b\bar{q}_1(t) - b\bar{q}_2(t)) \tag{xiv-a}$$

$$q_2(t+1) = (1-\nu)\bar{q}_2(t) + \frac{\nu}{2b}(a - \bar{c}_1 - b\bar{q}_2(t)) \tag{xiv-b}$$

3. Numerical Simulation and Analysis

The simulation runs at two different strategies (homogenous as in x) and heterogeneous as in xiv-a and xiv-b) and the firm j with some effort on market share. This simulation is aimed to see how the rate of market share influences the production output for both firms in bifurcation map with control parameter as the rate of market share over production output.

The simulation is conducted using these following coefficients:

$a = 10$; $b = 0.5$; $c_1 = c_j = 3$
$c_2 = c_{-j} = 5$; $\nu = 0.65$; $a_j = 0.42$; $a_{-j} = 0.35$; $g = 0.01$
$q_1 = q_j = 3$; $q_2 = q_{-j} = 3$.

The first simulation is carried out for the firm with homogenous strategy and the second is for heterogeneous one.

As we know, the behavior of attractor on the bifurcation map in Figure 1 has different character from general bifurcation map that usually starts with stable condition before turns chaotic as it reaches the end. On the other hand, in this homogeneous strategy, bifurcation map is initiated with chaotic condition and then turns into stable condition, which is marked by a number of white windows assigning a finite numbers of point attractor.

Generally speaking, homogeneous strategy is dominated by chaotic condition at almost all of control parameter. The output production of the firm j increases over the firm $-j$. Even if the control parameter is at the maximum, that is, above 0.4, production output of firm $-j$ tends to reach zero, so the market is fully dominated by the firm j, and the firm j became the single player in this duopoly game, or in other words, monopoly. This can be seen from the dramatic degradation of production output of the firm $-j$ in interval [0.26,0.4].

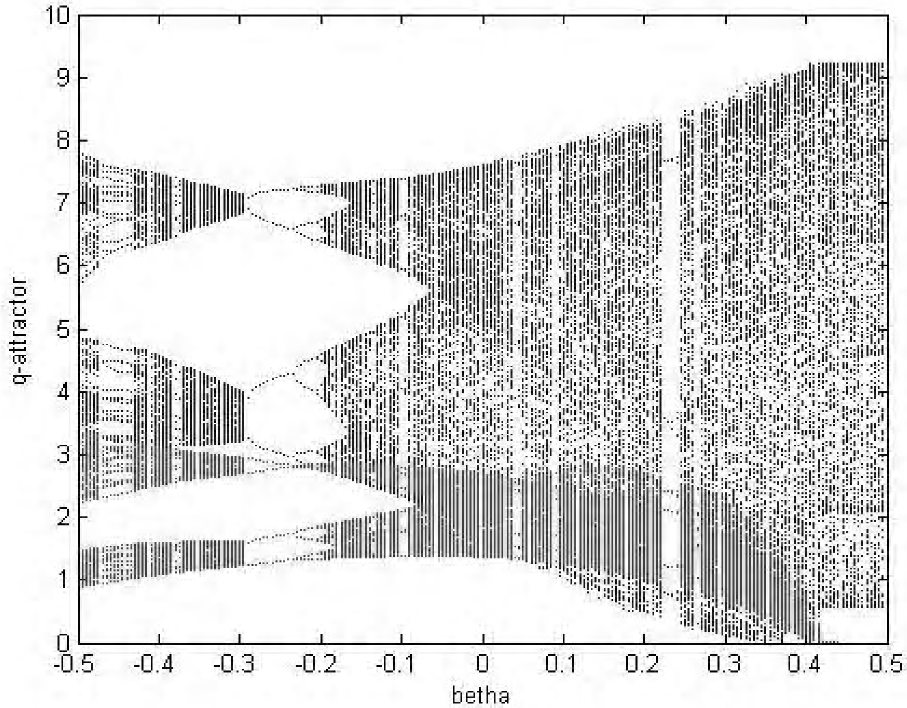

Figure 1 **Homogenous Bifurcation Map**

In homogenous strategy, condition dominated by chaotic pattern in almost all parameter control. Firm *j* bifurcation (dotted/upper) and firm *-j* bifurcation (solid/lower).

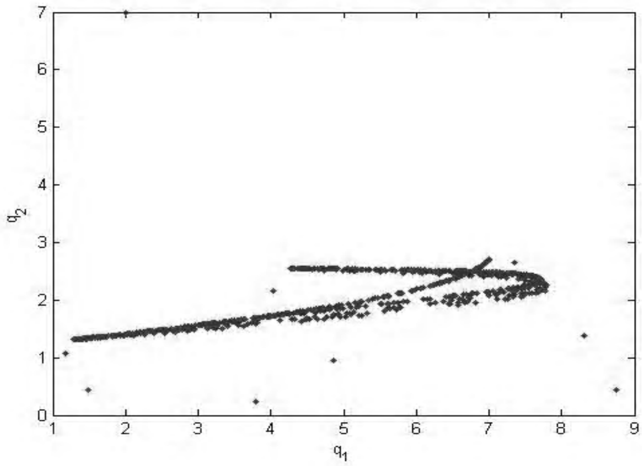

Figure 2 **Attractor on chaos for** $\beta = 0.07$

Attractor between output production of the firm *j* and the firm *-j*. Watch that almost all output production firm *j* grows twice from output production firm *-j*.

Before the eventual domination of the firm *j*, at high control parameter [0.1,0.4], an interesting phenomenon arises. In chaos, point attractors of the firm *j* are widening, approaching lower-right side of point attractors of the firm *-j*. This gives information that in certain interval, the presence of some effort on market share will decrease output production.

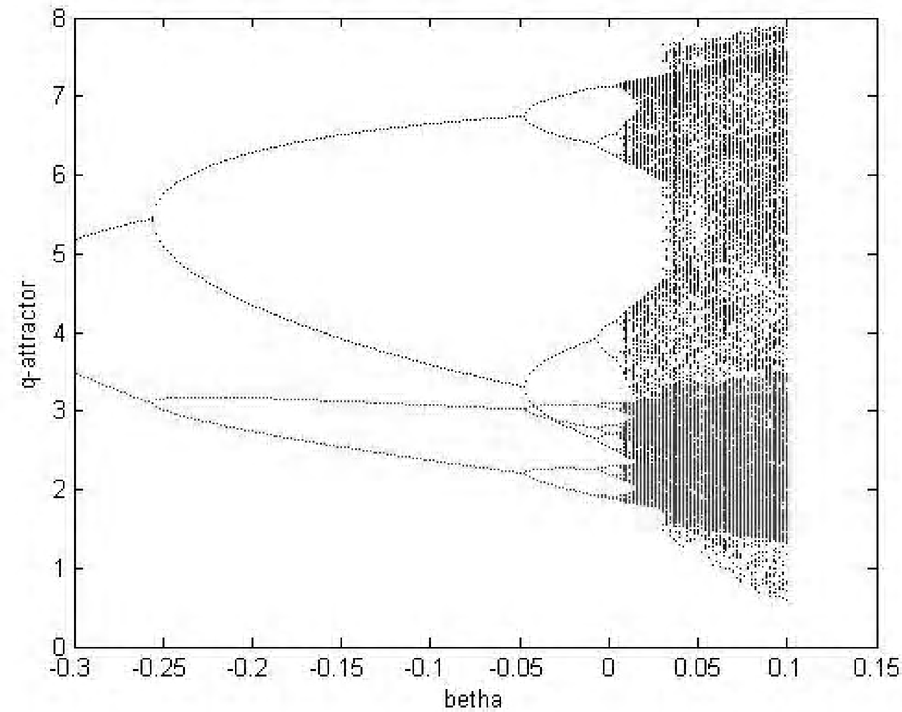

Figure 3 **Heterogenous Bifurcation Map**

In heterogeneous strategy, it begins with 1-point attractor for both firms and at control parameter 0.01 turns chaotic for both firms on output productions.

In the interval of control parameter [-0.2,0.4], the numbers of production output for both firms are the same. Yet this intensity is less than a quarter from all probability of point attractor.

The attractor in Fig. 2 occurs in chaotic condition at control parameter. It is interesting that in Fig. 2 the pattern of chaos differs in the random condition. From this pattern we can see that almost all of output production firm *j* are as twice as of the firm *-j*, except for production output below 1.5 where production output of firm *–j* can compete with the firm *j*. It also appears at some values beyond the pattern.

Different from homogeneous strategy, in heterogeneous strategy, as we see in bifurcation map, attractor starts with stable condition (finite point attractors) and turns to chaos. In homogeneous strategy, most of control parameters showing stable condition as compared to the domination of chaos condition at the previous simulation. Chaos occurred at control parameter near the maximum value of 0.1, that is 0.02. In general, the firm *j* with some effort on market share within the model can dominate at all of parameter control; however it does not take over the game to become monopoly.

It is an interesting fact that in chaotic condition, at control parameter greater than 0.02, the production output of firm *j* is the same with the output of firm *-j* with certain intensity approximately less than a quarter. And in this chaos with the certain intensity, less than 1/10 capacity of production of firm *j* is below the output production of firm *-j* .

The attractor of the firm *j* and the firm *–j* over output production forms showing more disseminating pattern than that in the homogeneous strategy. And similar with the previous

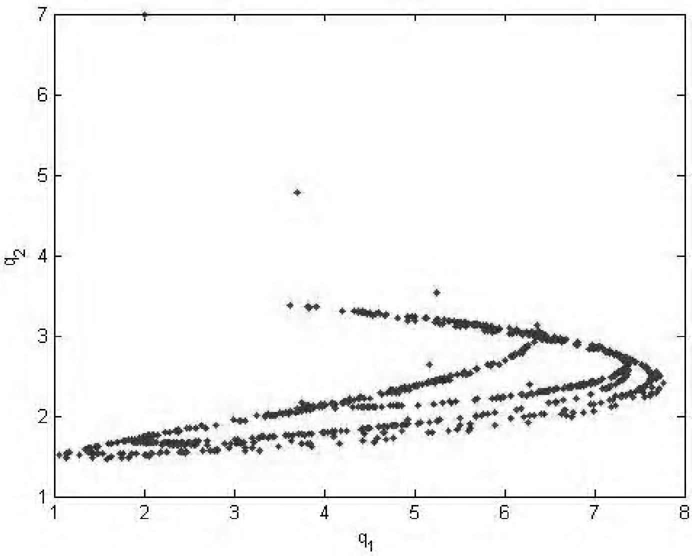

Figure 4 **Attractor in chaos for** $\beta = 0.056$

If we compare with homogenous strategy, this attractor is more disseminated, however, the output production of firm *j* remains twice of firm -*j*.

simulation, on production output less than 1.5, the firm -*j* still can make balance and even exceed its rival, although at production output more than 1.5, the firm *j* dominates.

4. Conclusions

This paper is based on the assumption that efforts which include innovation, cost on production and efficiency, would affect market share in duopoly game.

When both firms use bounded rationality, the bifurcation map is characterized by chaotic condition at the initial and finally dominated by chaos in almost all of control parameter. On the contrary, for the firms with heterogeneous strategy, bifurcation map started with the ordered condition and chaos only occurred at near maximum value of control parameter.

For homogeneous strategy, at high control parameter, or when influence of market share is very high over production output, duopoly competition finally turns into monopoly regime. It is shown by the slow decreasing of output production of the firm -*j* up to certain control parameter where it is no longer able to compete against the firm *j* at control parameter . This condition does not occur in heterogeneous strategy.

For both strategies, attractor in chaotic condition (Figure 2 and 4) is showing that the production output of the firm *j* and the firm -*j* have similar behavior and pattern: the firm *j* dominates over the firm -*j* (production output of firm *j* reach twice of firm -*j*). However, for small output production (less than 1.5) the firm - *j* can make balance over the firm *j*.

Further works wil be concentrated on time delay for market share and management structure level for each firm in duopoly game. Lenox (2002) have developed the model of innovation discovery (as some effort on market share) and efficiency with information transfer, organization design and knowledge capacities.

6. Acknowledgements

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8. Appendix

a=10; b=0.5; $c_1=c_j$=3.1

$c_2=c_{-j}$=5; v=0.65; α_j=0.35; α_{-j}=0.42; γ = 0.01

$q_1=q_j$=3; $q_2=q_{-j}$=3

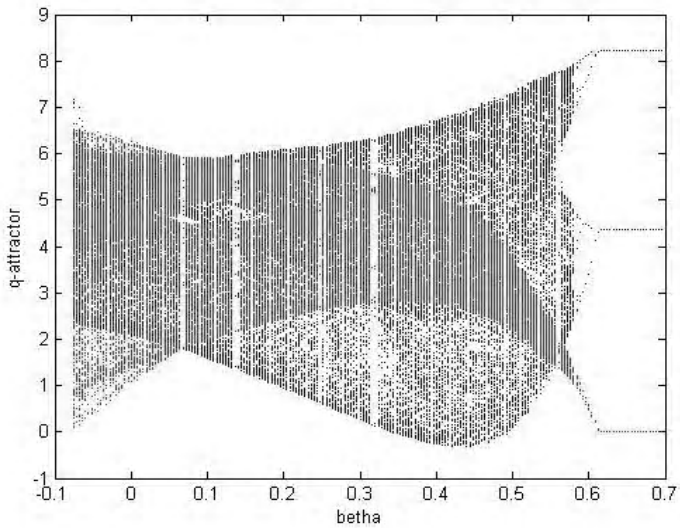

Figure 5

Homogenous strategy, stable on three points attractor

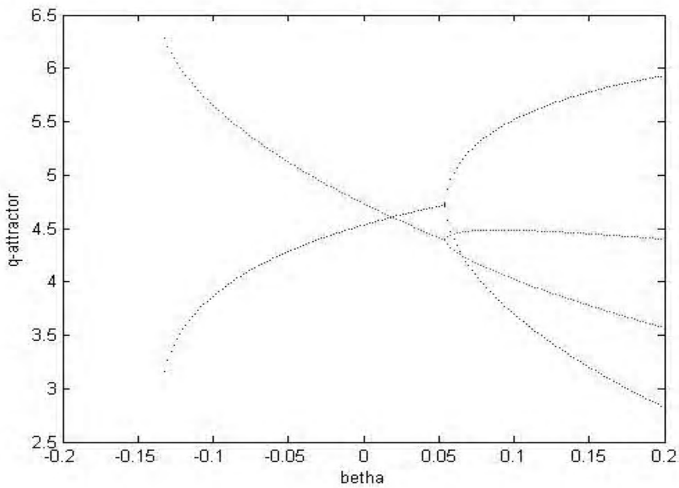

Figure 6

Heterogeneous strategy, stable condition on all parameter control