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Conflict in The Spatial Lotka Volterra using Prisoner's Dilemma

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Abstract

Conflict in Lotka-Volterra model assumed to be competition between agents. This paper tries to develop type of Generalized Lotka Volterra (GLV) to become further spatial type and interaction between agents modeled in prisoner's dilemma game theory. Three types of the spatial LV model were simulated in this paper. They are: pure spatial LV model, spatial LV model by Prisoner's Dilemma with the agent strategy to seek the maximum payoff from its neighbor, and spatial LV model by PD with random strategy respectively. The simulation resulted power-law distribution that emerges within the three spatial LV models. The existence of agent's learning process will push or block gap-attaining between agents, meanwhile number of rounds will accelerate the attaining of power law.

Keyword: conflict, Generalized Lotka Volterra, Spatial, Prisoner Dilemma, Power-Law.

1. Introduction.

At the beginning, Lotka Voltera model was used as a simple approach to two species which were inter-dependent within the food-chain, e.g. predator-prey between snakes and mice. Furthermore, this model is not only limited in the use of modeling ecological system, but also used widely in other fields. For example in economy Godwin modeled Marxian economic system as predator-prey, relationship between mining companies with the society surrounding (Hariadi, 2003). Classic LV model has been largely modified, e.g from the swarm over LV model which means involving n-species to modification over Lotka Voltera characteristic equation itself, e.g differentiation from continue model to discrete, even from deterministic to stochastic.

The main point in LV model is the existence of competition between two different species in which their interaction, one will play as predator, the rest are prey. In this paper, LV model is used as one of models to approach conflict cases. Conflict in LV model is assumed as competition form of different species, in which one play as predator, the rest as prey. Each species needs other species in order to survive. Let's take an example: conflict between mining company and its workers is modeled as competition between the two different agents (Hariadi, 2003). Another example is conflict between capital owner and workers, modeled by Goodwin as competition between number of workers with wages change (Goodwin).

Theoretically, the study of LV has been furtherly developed. LV model which involves many agents has been analyzed by establishing stabile evolutionary ecology concept (Cressman & Garay, 2002), global bifurcation structure (Kan-on, 2002), stochastic LV (Solomon, 1998), Generalized LV (Solomon, 1998), and power-law occurrence (Solomon, 1998). An adequately important thing in the study of complex system is power-law characteristic, which bridges simple behavior microscopically at individual level and macroscopic phenomena in collective level (Solomon, 1998).

This paper aims on giving different approach in using GLV model: first, GLV approach which has spatial character. Second, interaction among agents in LV needs to give opportunity to choose one role. In this way an agent does not only play as prey, but at certain time it can be a predator. Even predator or prey should receive opportunity to change its level of predatory or prey, and all agents involved respectively are not considered as equal. The latter proposition needs to use model in game theory.

This paper is structured as follow: section two gives initial definition about GLV and construction of spatial LV model using the game theory, in which this paper uses prisoner's dilemma. Section three is simulation and analysis. Simulation consists of three sections, i.e.: first, pure spatial LV model, second, spatial LV model using prisoner's dilemma with agent's strategy to find maximum payoff from its neighbor, third, LV model using PD with random agent's strategy. The final section of the paper is closed with discussion and conclusion.

2. Notation and Model.

Following notation of the Generalized Lotka Volterra for n-agent, let us assume w_i(t) as an expressing wealth value of agent *i* at time *t*, hence total wealth value of overall agents are:.

$$w(t) = w_1(t) + \cdots + w_N(t)$$

and the General LV model is

$$\begin{aligned} w_i(t+1) &= \lambda(t) w_i(t) + a(t) w(t) - c(t) w_i(t) w(t) \\ w_j(t+1) &= w_j,\ j = 1, ..., N;\ j \neq i \end{aligned}$$

With $\lambda(t)$ representing autocatalytic in individual level, *a*(*t*) is autocatalytic in community level that is advantage obtained as a result of interaction with the community, and *c*(*t*) is environment saturation level as a consequence of interaction and competition between agent *i* and overall agents, with $\lambda(t)$ is distributed $\prod(\lambda)$. The model gives a full opportunity for each agent to interact with other agents. The characteristic occurs is power-law between P(w), number of agents who own wealth w, *a*(*t*), and *c*(*t*) constant (Solomon, 1998).

$$P(w)dw \sim w^{-1-\alpha}dw, (\alpha > 1)$$

The contribution of this paper is to model generalizing LV with limitation on inter-agent interaction, means agent doesn't interact with all other agents in a system, interaction occurs only limited to neighborhood. This paper uses linkage model between an agent and other 4 agents. So, wealth of linked agent-*i* is:

$$w_{n_i}(t) = w_{-i_1}(t) + w_{-i_2}(t) + w_{-i_3}(t) + w_{-i_4}(t)$$

with $w_{-i_1}(t), w_{-i_2}(t), w_{-i_3}(t), w_{-i_4}(t)$, each represents wealth of an agent which is linked to agent-*i* in *t* time, commonly named neighborhood wealth, so the GLV becomes:

$$w_i(t+1) = \lambda(t)w_i(t) + a_i(t)w_{n_i}(t) - c_i(t)w_i(t)w_{n_i}(t)$$

$a_i(t)$ represents profit level gained by agent-*i* when having interaction with her/his neighbor agents, while that $c_i(t)$ represents competition level between agent-*i* and her/his neighbor which causes decreasing in wealth gained by agent-*i*.

Interaction between agents here is modeled with game theory using prisoner's dilemma. Prisoner's dilemma is the most popular non-zero game type, especially in economy, politic, or nuclear conflict between USA and Russia.

Generally, payoff used in PD can be normalized to (Billard, 1995):

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{bmatrix} 0.5, 0.5 & -1, 1 \\ 1, -1 & -0.5, -0.5 \end{bmatrix} \end{array}$$

Furthermore, game theory using PD has been developed into evolutionary type, interaction between agents performed several times, and each agent use his smartness to decide which strategy to gain profitable payoff. Including in this model is the existence of learning process from agent so that for each iteration in PD, agent equipped with learning process will advantage the past as her/his basic consideration in taking decision. Model used in this PD is not limited deterministic, yet begins to adopt stochastic models, i.e.: Billard (Billard, 1995) and Eriksen (Eriksen et al, 2003).

PD continues to develop from only just a game involving 2 agents in 1 round to game theory which involving n-agents in rounds with quasi non-limit. The characteristic of PD also shows specification that is how the interaction pattern between agents inolved, is an agent linked to overall agents, or only undergo interaction with limited neighborhood.

Several developments in PD is the utilization of continuity element which gives freedom to take decision not only C or D, but decision between C and D (C<decision<D)

is also included in decision choice, C and D have certain degree, therefore will occur different cooperative degree, as performed in (Nowak et al, 1999).

PD iterations give interesting challenge which is how attitude as dominant strategy within all iterations emerges so that characteristic of ALLD is not a good choice. PD iterations models be it involving 2 agents or more is used to investigate the emergence of altruism, to investigate altruism in 3-PD which involving Nash equilibrium (Arce, 1994), or to answer question whether cooperative is natural.

PD Iteration type needs a model that is able to describe how the learning process gained from agent. This learning model from agent is giving choice of how characteristic of agent involved, whether pure rational or just partly, rationality in which determined by how much information gained by agent. Another thing can influence learning model is the length of memory that the agent has; this can give different influences in determining strategy in PD.

Deterministic and stochastic model used in modeling learning process from agent, e.g in continuous PD (Nowak, 1999), there used deterministic function $y = y(x_i, x_{i-1}, x_{i-2}, \cdots)$ with $\{x_i\}$represents opponent's strategy and y is response or answer made by opponent. Stochastic approach performed by (Billard) which changed PD payoff function to probability form, the existing payoff meant agent opportunity for gaining reward (in this approach, there will show up *mixed strategy* terminology which is differ from pure strategy). Stochastic approach emphasizes on agent's irrationality in taking decision.

Payoff showed in this game can be used to determine $a_i(t)$ and $c_i(t)$. Suppose for game theory, $G = \left(N, (C_i)_{i\in N}, (u_i)_{i\in N}\right)$, with N is players set, C_i is players' strategy set, and u_i is payoff function. In this paper, a player only interacts with her/his 4 near neighbors, so strategy space for player changed from:

$$C = \underset{i\in N}{\times} C_i \text{ becomes } C = \underset{i\in n_i}{\times} C_i$$

With n_i represents linkage set between players and agent-*i*, or neighbor-*i* set. Then, for $c \in C$, u_i represents payoff gained by agent-*i* if choose strategy c in mapping.

$$u_i : C_i \times C_{-i_1} \times C_{-i_2} \times C_{-i_3} \times C_{-i_4} \to R^4$$

with $C_{-i_1}, C_{-i_2}, C_{-i_3}, C_{-i_4}$ represent each agent *i* neighbor's strategy, denoted as C_{-i},

$$u_i(C_i, C_{-i}) \begin{cases} a_i, \phi(C_i) > 0 \\ c_i, \phi(C_i) < 0 \end{cases}$$

$$\phi : R^4 \to R$$

So in *t* time wealth of agent-*i* is

$$w_i(t+1) = \lambda(t)w_i(t) + a_i w_{n_i}(t) - c_i w_i(t) w_{n_i}(t)$$

Strategy is a decision planning which will be taken by agent in next interaction. In this paper, the strategy will be taken appropriately to maximum payoff gained by neighbor, and agent will change her/his strategy appropriately to maximum strategy of neighbor with equal opportunity ratio between her/his wealth and neighbor's.

3. Numerical analysis & Discussion

The goal of simulation performed in this paper is to seeing the agent behavior that interacts in spatial Lotka Volterra model, farther will be seen how the behavior would be if among the interacting agents have the ability to change her/his position on predator or prey or change her/his degree of predatory or prey. In the simulation, interaction of agents is modeled with game theory, prisoner's dilemma.

The simulation involved 10000 agents in matrix 100x100 where each agent connected with four neighbor agents and each agent only interacted with her/his neighbor. One round defined with time required by agent for having interaction with all its neighbor members. Hencefoth, within one round means the time required by agent for interacting with other four agents, in this case her/his neighbors.

There are three kinds of LV model simulated in this paper, i.e.:

1. The pure spatial LV model, where each agent interacting in this model interacts with constant predator-prey degree.
2. The spatial LV model with PD in which the strategy of imitating the neighbor's maximum payoff with the probability based on her/his wealth compared with her/his neighbor's wealth.
3. The spatial LV model with PD in which the strategy is having random character of random, means the agent will change her/his strategy in each round.

In the first model simulation, all agents were assumed to have the same ability of acting as predator and prey with $a_i(t)$=0.01 and $c_i(t)$=0.1, and each agent has uniqueness for his wealth in the beginning simulation.

Differ from the first simulation, where each agent assumed to have the same ability of interacting with her/his neighbor, in the second simulation, each agent has different ability in determining level of her/his predator and prey. This is determined by each payoff obtained in prisoner's dilemma. Henceforth, in the beginning of the second model simulation, besides requiring initial wealth also required initial strategy.

For every round, each agent in this second simulation will count her/his payoff obtained from interaction with her/his neighbor and compare it with neighbor's payoff. If the highest payoff she/he obtained from interaction with her/his neighbor is smaller than the highest payoff obtained by her/his neighbor whom she/he interacts, hence agent will change her/his strategy with the probability that proportionate to how big agent's wealth value compared with her/his neighbor's.

The third model simulation has nearly the same procedure with the second model simulation. The differing point is the way of taking strategy where in the third model simulation, the agent will take her/his strategy at random without reckoning payoff obtained by her/his neighbor and also by payoff he got from the previous rounds.

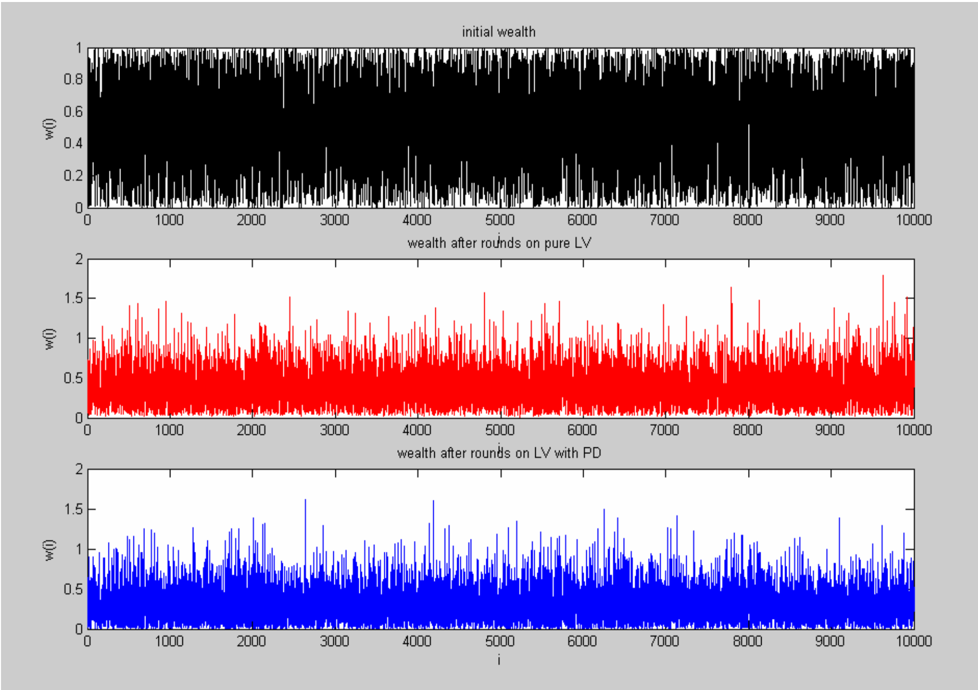

Figure 1
The three graphs above showing properties value of every agent, the toppest graph represents the initial wealth of every agent, the middle picture shows the properties value after 10 rounds from first simulation model, and the bottom graph showing properties value after 10 rounds model by the second simulation

The simulation result shown above showing the changing wealth value of every agent at first simulation model and the second one. At early stage, the agent's wealth distrtributed uniformly, interaction between agents in both model LV (first model simulation and second model) will change the wealth of every agent, and at round 10 of the simulation yielded several agents whose wealth is far exceed the other agent's. This phenomenon appeared (in the middle and the bottom picture) with the emerging of some agents owning wealth level above others, in which on the beginning of simulation every agent's wealth did not exceed of one..

At Figure 1 above, it is quite difficult to differ the first model simulation result (the middle picture) with the second one (the bottom picture in figure 1), both model simulation result almost similar. To find the difference is requiring different depiction from both model simulations. The estimation result of probability distribution of both model simulation is given in Figure 2. As shown in the picture, that the second model simulation result showing the higher and slimmer distribution compared to the first model simulation. This point informs that in the second simulation model, number of agents owning little wealth/poor (less than the value of first intersection between first model distribution with the second one) more numerous compared to the first model. It is equal to say that there are more agents owning little wealth at second model compared to the first model.

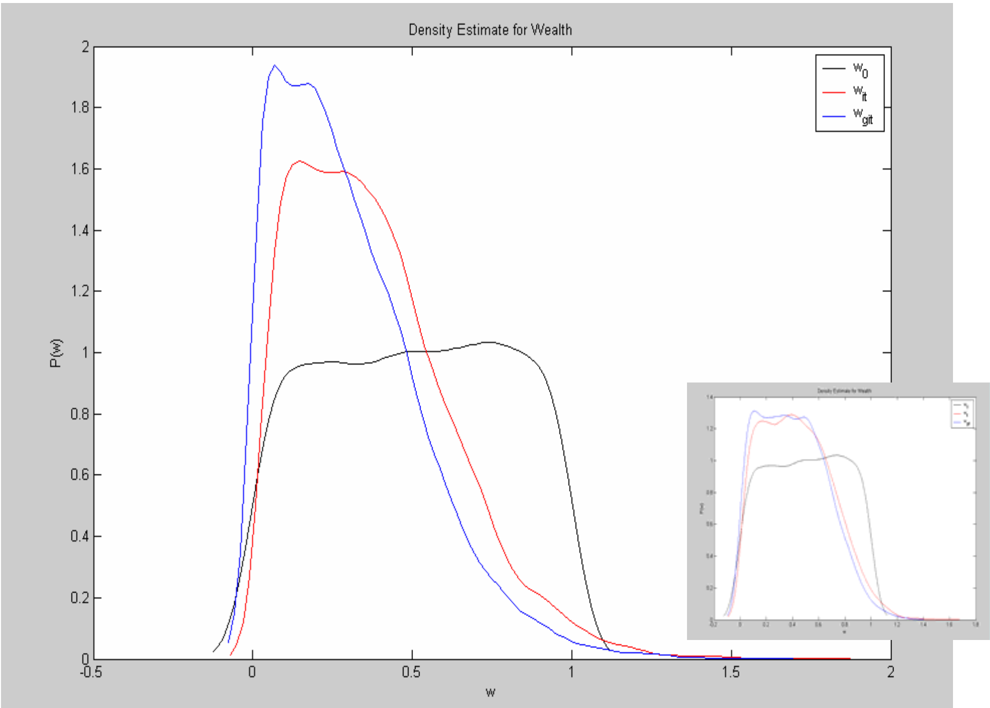

Figure 2

Each color represents the form of estimated distribution, black color represents the estimated distribution form from initial wealth, red is for estimated distribution of agent's wealth after 10 rounds in the first simulation model, blue color represents estimated distribution form of agent's wealth after 10 rounds in the second simulation model. Inset picture represents estimated distribution from both model simulation at 5 rounds. Black color in the picture represents the form of estimated distribution of initial wealth.

With the existence of distribution transformation in picture above means there is a change, in this case is the amount of wealth. The number of "poor" agents progressively increase by the increasing rounds. Meanwhile the form of the simulation model determining how many agents turn " poor". For example the first simulation model compared to the second simulation model will cause the difference of the number of "poor" agents. How about the number of "rich" agents? The number of "rich" agents in distribution form above is decreasing by the increasing round. It is the same to say that the simulation performed above is yielding a large number of "poor" agents and yielding fewer "rich" agents (the inset picture). At round five, eventhough change of wealth distribution occurs, compared to the initial wealth distribution, yet, the form of the first distribution model simulation with the second distribution model showing no difference.

In general, the result of different simulation model of the first and the second one indicating the existence of opportunity for agents to do the transaction or opportunity to change the degree of predator and prey level for agents, in which this simulation is modeled with Prisoner's Dilemma and this situation will be exploited by agents to enlarge the profits. This matter reflected in the decreasing of the number of "rich" agents in the second model simulation compared with the first one. The model in the second simulation open wide opportunity for all agents to gain advantages altogether fom the interaction in PD model, if this phenomenon occurred, all agents will choose the ALLD strategy. If this strategy is selected, there would be a decreasing amount of wealth or rising degree of prey for each agent, with the value of $a_i(t)=0$ and $c_i(t)= - 2x0.1$ for all agents. This means that wealth change only resulted from the wealth reduction, as the prey effect is high..

The following simulation aims on monitoring influence of strategy to the wealth distribution. To find out the result is requiring simulation as in the second simulation model but with random strategy. The result of this simulation is given in Figure 3.

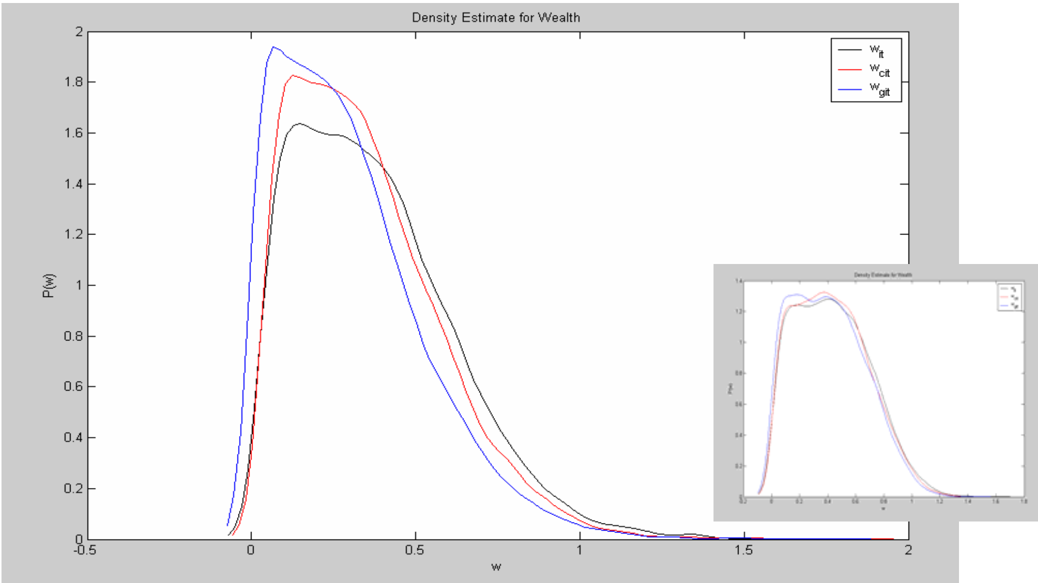

Figure 3

The previous two simulations are compared with the third model simulation that is the simulation of spatial LV model with PD using random strategy. The figure above represents form of estimated distribution within 10 rounds. The inset picture represents the simulation result at round 5. The red line in the picture represents random strategy.

The third simulation giving result that explains the role of strategy in a holistic fashion. The result of simulation with random strategy indicating that this strategy has higher and slender distribution compared to the first simulation model (pure LV), but has the lower and fatter distribution compared to the second simulation model. In other words, simulation with random strategy made a number of “poor” agents increases and “rich” agents decreases compared to the first simulation model, but it has inverse character compared to the second model simulation. This simulation also indicating that the number of rounds also determines the following simulation distribution difference.

4. Conclusion And Discussion

The Lotka Volterra model represents the model used to depict the competition among number of agents. The result of this competition is the emerging of predator and its victim (prey). Further, this Lotka Volerra model experiences many modifications, from addition of amount of agents in involved interaction up to the general model which means involving a number of *n* agents in its interaction. Modification also subjected to the nature of the Lotka Volterra equation itself, e.g. the change from continue to discrete model, even from deterministic to stochastic model.

Sorin Solomon has investigated the relation of the Lotka Volterra model with the nature of power law [Solomon,1998]. In another fashion, this paper is proposing and simulating two matters: first, interaction of agents in Lotka Volterra model localized. Second, interaction of each agent in first proposal needs an addition of learning ability, or in other words giving opportunity to every agent to chose the beneficial role in predator-prey model. The game theory modedel used in this simulation is Prisoner’s Dilemma, this is because PD gives payoff for cooperation or defect, whereas in LV model lies coefficient of predator and prey which showing advantage level as the effect of the existing of cooperation and the loss level as the effect of interaction.

The result of simulation with the two proposals yielding the equal nature with the classic LV model that is the appearance of power law character. If the classic model requires many rounds to reach the power law, the spatial model requires more short rounds. In further, the presence of game theory will provide opportunity for every agent to improve her/his strategy in a round, eventhough this opportunity will also give risk for agent to be exploited if she/he choose the wrong strategy. Difference result of simulation with pure LV model and the LV model with game theory, lies in how fast the occuring of difference among wealth agent. Simulation result gives information that for the agent with learning ability, will cause quicker difference attainment (gap) or quicker exploitation. However, it is of no possibility that the existence of learning process will affect to slow down time to reach difference (gap) among agents.

Different result of simulation of the first model (the pure spatial LV model) and the second model (pure spatial LV with game theory) exhibiting the presence of opportunity for agent to do transaction or opportunity to change the rate of predator and prey to her/his neighbors which will be used by agents to enlarge the profit. This point is strrongly denounced by the result obtained in the simulation of the third model in which the agent strategy has the character of random. It is obvious that agent with the random strategy yielding higher and fatter distribution compared to distribution form of the second strategy where the more rational agent (simulation of second model) having more character of exploitation. Another important thing which will influence the form of wealth distribution is the number of round. Be it agent is provided or not with learning ability, the number of rounds will speed up the reaching of power law.

5. Further Works

Interaction between agents in this spatial model is still limited to simple form where one agent is connected with her/his neighbors or the four closest agents. It requires further development on how if the member of her/his neighbors is multiplied, even to a condition where the number of neighbors is different one another.

For the model with game theory requiring the development to Nash equilibrium, in order to answer the question: whether nature of power law in LV model emerges when agent interaction in game theory stay in the Nash Equilibrium?

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References

1. Billard, Edward A.(1995). *Adaptation in a Stochastic Prisoner's Dilemma with Delayed Information*. BioSystems, Elsevier Science Ireland Ltd.
2. Boucher, D.H.(1995). *Lotka Volterra Models of Mutualism and Positive Density-Dependence*. Ecological Modeling. Elseiver Science Publisher B.V.
3. Delahaye, J. P. et al.().*Studies on Dynamic in the Classical Iterated Prisoner's Dilemma with Few Strategies is there any chaos in the pure dilemma?.*
4. Elsadany, A.A.&Agiza, H.N.(2003).*Chaotic dynamics in nonlinear duopoly game with heterogeneous players*.
5. Henriksen, C. et al.(2003). *Probabilistic learning Automata and The Prisoner's Dilemma.*
6. Hariadi, Yun.(2003). Perusahaan Tambang, SDA, dan Masyarakat dalam Lotka Volterra yang dimodifikasi.Working Paper BFI
7. M, Daniel G. Arce.(1994). *Stability Criteria for Social Norms with Aplications to the Prisoner's Dilemma*. The Journal of Conflict Resolution, Vol. 38, No. 4.
8. Matsui, Akihiko. & Gilboa, Itzhak.(1991). *Social Stability and Equilibrium*. Econometrica, Vol.59, No.3.
9. Muhlenbein, Heinz. et al.(2002), *Evolution of Cooperation in a Spatial Prisoner's Dilemma*. arXive:cond-mat/0211605v1
10. Nowak, Martin A.& Wahl, Lindi M.(1999). *The Countinous Prisoner's Dilemma:I Liniear Reactive Strategy*. J. Theor. Biol.
11. Sen, Arunava. & Abreu, Dilip.(1991). *Virtual Implementation in Nash Equilibrium*. Econometrica, Vol. 59, No. 4.
12. Solomon, Sorin.(1998). *Generalized Lotka-Volterra (GLV) Models*. Econophysics, Kluwer.

13. Solomon, Sorin(2001). *Power Law of Wealth, Market Order Volumes and Market Returns.* Physica A 299 188-197. Elseiver.
14. Solomon, Sorin.(1998). *Stochastic Lotka-Volterra system of Competing Auto-catalytic Agents Lead Geneically to Truncated Pareto Power Wealth Distribution, Truncated Levy Distribution of Market Returns, Clustered Volatility, Booms and Crasches*. Kluwer Academic Publishers.
15. Stephen, D. W. & Nishimura, K.(1997). *Iterated Prisoner's Dilemma: payoff variance*. J. Theor. Biol
16. Szidarovszky, F. & Forgo, F. *On Relation Between the Nash Bargaining Solution and the weighting method*. European Journal of Operational Research.
17. Toke, C. & Szabo, G.(1997). *Evolutionary Prisoner's dilemma Game on square Lattice. Physical Review* E. Vol. 58, No.1.